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PHYSICAL REVIEW.

A STUDY OF THE TRANSMISSION SPECTRA OF CERTAIN SUBSTANCES IN THE INFRA-RED.

By Ernest F. Nichols.

WITHIN a few years the study of obscure radiation has been greatly advanced by systematic inquiry into the laws of dispersion of the infra-red rays by Langley,[1] Rubens,[2] Rubens and Snow,[3] and others. Along with this advancement has come the more extended study of absorption in this region. The absorption of atmospheric gases has been studied by Langley[1] and by Ångstrom.[4] Ångstrom[5] has made a study of the absorption of certain vapors in relation to the absorption of the same substances in the liquid state, and the absorption of a number of liquids and solids has been investigated by Rubens.[6]

In the present investigation, the object of which was to extend this line of research, the substances studied were: plate glass, hard rubber, quartz, lamp-black, cobalt glass, alcohol, chlorophyll, water, oxyhæmoglobin, potassium alum, ammonium alum, and ammonium-iron alum.

[1] Report on Mt. Whitney Expedition, Profess. Papers, U. S. Signal Service, XV.
[2] Annalen der Physik und Chemie, N. F. XLV., p. 238.
[3] Annalen der Physik und Chemie, N. F. XLVI., p. 529.
[4] Bihang till K. Svenska Vet.-Akad. Handlingar, Band 15, Afd. 1, No. 9.
[5] Ofversigt af Kongl. Vetenskaps-Academiens Forhandlingar, 1890, No. 7, Stockholm.
[6] Annalen der Physik und Chemie, N. F. XLV., p. 258.

Apparatus Used.

The galvanometer used was similar in many respects to the instrument described by B. W. Snow,[1] though designed and built independently and used several months before the description referred to appeared. The aggregate weight of the moving parts of the galvanometer was 48 mgs. Throughout the work the four coils were connected in multiple with a resistance of 9.3 ohms. The sensitiveness was varied from time to time for convenience, but for the measurement of energy in the weaker parts of the spectrum deflections corresponding to currents of 10^{-10} ampère were read.

In circuit with the galvanometer was a thermopile, made up of ten bismuth-antimony junctions arranged in a line 15 mm. long, the line being filed until the exposed surface was less than a millimeter in width. The arrangement of the entire apparatus for the study of the infra-red spectrum is given in Fig. 1. The whole is a modified spectrometer. L is a fifty-volt Edison incandescent lamp, the spectrum of which was used as a basis for determining the transmission of the various substances; *f's* are diaphragms, *D* the prism, *ABC* a device for automatically maintaining minimum deviation, *E* a track on which layers of the substances under examination could be passed between the lamp and the first lens, and *T* is the thermopile, interchangeable with the cross-wires of the eyepiece of the observing telescope. The apparatus was arranged in two rooms; the lamp *L* and the track *E* in one, and the remaining parts of the spectrometer in the other. A small hole in the partition, which could be closed at will by means of a shutter, served to connect the two rooms. The room containing the thermopile and prism

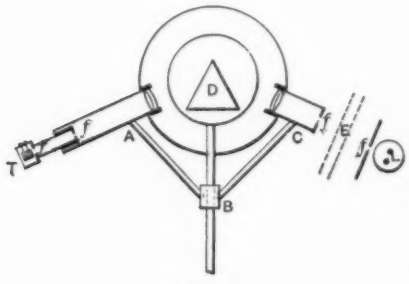


Fig. 1.

[1] Annalen der Physik und Chemie, N. F. XLVII., p. 213.

was kept dark, and all draughts excluded. It was entered only once or twice a day to change the spectrometer setting. The glass lenses of the spectrometer were retained. It is easily seen that their absorption could not enter as an error in the present investigation; for if the energy of the lamp at any wave-length be taken as unity, and the lamp be allowed to shine on the thermopile so that it receives this wave-length only, with nothing between but a system of plates transmitting a percentage a of the energy they receive, the galvanometer deflection will then be proportional to a or ka. If now another body, which transmits a percentage b of the energy received by it, be placed between the lamp and the plates, then the energy which reaches the thermopile will be ab, and the galvanometer deflection kab. The ratio of the second deflection to the first is b, the percentage of transmission of the body in question.

A carbon bisulphide prism served for dispersion; that substance was selected because it is known that carbon bisulphide at a temperature of fifteen degrees follows the well-known Cauchy formula for dispersion: $N = a + \frac{\beta}{\lambda^2}$ where N is the refractive index and λ the corresponding wave-length.

Rubens[1] has investigated experimentally the law of dispersion of bisulphide of carbon out to wave-length 2 μ, and has found it to agree very closely with this law, and further, from the near agreement between the square root of the dielectric constant and the index of refraction for infinite wave-lengths computed from the Cauchy formula, it is probable that the dispersion follows Cauchy's law throughout the whole spectrum. The carbon bisulphide used in the present study was obtained by double distillation, and the variation of its indices of refraction with temperature was carefully studied at various wave-lengths. The results were found to agree very closely with similar results obtained by Wüllner.[2]

A comparison of the two is given in Table I.

[1] Annalen der Physik und Chemie, N. F. XLV., 1892, p. 256.
[2] Annalen der Physik und Chemie, Band CXXXIII., 1868, p. 16.

TABLE I.

C. LINE.			F. LINE.		
Temperature.	Wüllner.	Nichols.	Temperature.	Wüllner.	Nichols.
18.2		1.6202	19.25		1.65335
18.25	1.6198		19.3	1.6530	
18.55	1.6196		19.42		1.6532
18.58		1.6198	19.6	1.6530	
18.6	1.6196		19.70	1.6529	
18.91		1.6197	19.76		1.6530
19.0	1.6193		20.18		1.6528
19.10	1.6192		20.25	1.6525	
19.14		1.6194	20.5	1.6522	
19.45	1.6189		20.54		1.6524
19.46		1.6192	21.08		1.6520
20.15	1.6183	1.6186	21.1	1.6517	
20.55	1.6181		25.27		1.6485
21.80		1.6181			
22.1	1.6169				
22.32		1.6170			
22.83		1.6166			
23.02		1.6164			
23.4	1.6159				
24.20		1.6156			
25.24		1.6147			
26.08		1.6142			

The temperature change in the indices of refraction is seen to be sufficiently large to make it necessary to correct all observations for temperature. The temperature coefficients of indices of refraction for the various Fraunhofer lines are given in Table II.

If N_t is the index of refraction for a wave-length λ at a temperature $t°$, and a is the temperature coefficient for that wave-length, then the index of refraction $N_{t'}$ for $t'°$ will be $N_{t'} = N_t - a\,(t' - t)$.

The temperature coefficients gradually diminished as the wave-length increased through the visible spectrum, and a similar decrease was assumed through the infra-red. In the temperature corrections made to observations, $t' - t$ was never greater than 3°. The temperature taken for the standard to which all observations were reduced was 23°.5. The prism was calibrated by the

TABLE II.

	A		C		D	
	Range	a	Range	a	Range	a
Wüllner [1]			$t = 7°$ $t' = 23°$	.00078		
Nichols	$t = 20°$ $t' = 25°.4$	.000759	$t = 18°.2$ $t' = 26°$	.000763	$t = 19°.8$ $t' = 26°.1$	.000768
	F		G		Line beyond G, N 23°.5 = 1.6782	
	Range	a	Range	a		
Wüllner	$t = 7°$ $t' = 23°$	.00082	$t = 7°$ $t' = 23°$	.00085		
Nichols	$t = 19°.2$ $t' = 26°.2$	.000805			$t = 21°.2$ $t' = 26°.2$	.000858

Fraunhofer lines, and the Cauchy constants computed. The equation to the calibration curve of the prism was $N = 1.5772 + \frac{.01681}{\lambda^2}$. The values $a = 1.5772$ and $\beta = .01681$ were obtained by observing the indices of refraction for the Fraunhofer lines *b*, *C*, *B*, and *A*, and substituting these indices in turn with their corresponding wavelengths into the general formula $N = a + \frac{\beta}{\lambda^2}$.

The indices of refraction observed and computed were :—

TABLE III.

	Observed.	Computed.
b.	1.6398	1.6398
C.	1.6156	1.6162
B.	1.6123	1.6128
A.	1.6063	1.6063

The prism on two angles of its base carried conical points. The point beneath the refracting angle fitted into a corresponding

[1] Annalen der Physik und Chemie, Band CXXXIII., 1868, p. 16.

pit; the other point into a groove in the table of the spectrometer. This device made it possible to take the prism off the spectrometer to allow a zero reading at any time, after which the prism could be accurately replaced. For calibration the ordinary spectrometer slit was used with an eye-piece cross-wire, but in the energy measurements, in order to get an intense source, a straight portion of the filament of the incandescent lamp was substituted for the slit, and the rest of the filament screened by diaphragms. To change from the slit to the filament, and from the cross-wire in the eye-piece to the thermopile, without disturbing the calibration, the spectrometer was set at zero reading, the eye-piece carefully focussed on the slit and the observing telescope clamped. The slit tube was then removed and the lamp filament put in the exact place where the slit had been, as determined by its image in the eye-piece. Then the eye-piece was taken out and a slit a millimeter wide was put in the plane of the cross-wires and moved from side to side until the image of the filament fell in the middle of the slit. The line of junctions of the thermopile was fitted accurately to this opening. After these changes the prism could be replaced on the spectrometer, the telescope carrying the thermopile could be unclamped, and by means of the calibration curve the thermopile could be set to receive radiations of any desired wave-length.

The width of the line of thermo-junctions in angular measure was 10′. Because of this width and the increasing condensation of rays by the prism as the wave-length increased, measurements beyond $\lambda = 2.^{\mu}6$ were of little value. The wave-lengths given in the tables correspond to the wave-lengths received by a line through the middle of the face of the thermopile.

Method of Observation.

Before beginning a set of readings, the galvanometer circuit was closed once for all. When the galvanometer needle was quiet, the shutter between the lamp and prism was opened, and the lamp allowed to shine on the thermopile until the needle had reached the end of its first swing: the shutter was then closed and the

return swing observed. The mean of the reading from which the needle started and the reading to which it returned was taken as the zero reading. Any uniform change in the zero point of the galvanometer during the ten or twelve seconds required to get the two swings was thus eliminated.

The percentage of energy of a given wave-length transmitted by any one of the substances was estimated as follows: 1st, The energy of the bare lamp was measured. 2d, The lamp energy transmitted through the substance. 3d, The energy of the bare lamp again. The ratio of the second reading to the mean of the first and third was taken as a transmission percentage. The use of the mean of the first and third for the lamp reading obviates the error which might otherwise enter from a uniform change in the constant of the galvanometer. Since the glow lamp was fed from a storage battery, the source of radiant energy was very nearly constant. From three to five determinations of this transmission percentage were made for each of the substances at all observed points. The mean of these readings is in each case the transmission percentage given in the tables. The probable error in the transmission percentages in no case exceeds $\pm 4\,\%$, and in most cases is less than $\pm 1\,\%$.

The Results.

Table IV. contains the transmission percentages in the infra-red spectra of a piece of French plate glass 8.3 mm. thick, and of a sheet of black hard rubber .2 mm. thick. The transmission spectra of these two substances were examined at nine different points and the corresponding wave-lengths are given in 10^{-6} meters at the heads of the columns.

TABLE IV.

λ	$0^{\mu}.775$	.862	.940	1.19	1.41	1.58	1.85	2.37	2.90
Plate glass . . .	82.4	76.9	77.5	71.7	83.7	82.1	82.5	75.1	76.7
Hard rubber . .	30.7	54.1	58.9	65.0	58.4	58.9	58.1	52.0	56.6

The results in this table are shown graphically in Fig. 2, in which wave-lengths are plotted as abscissæ, and the corresponding percentages of transmission as ordinates. The parallelism of the curves beyond $\lambda = 0^{\mu}.862$ seems to show that if the rubber

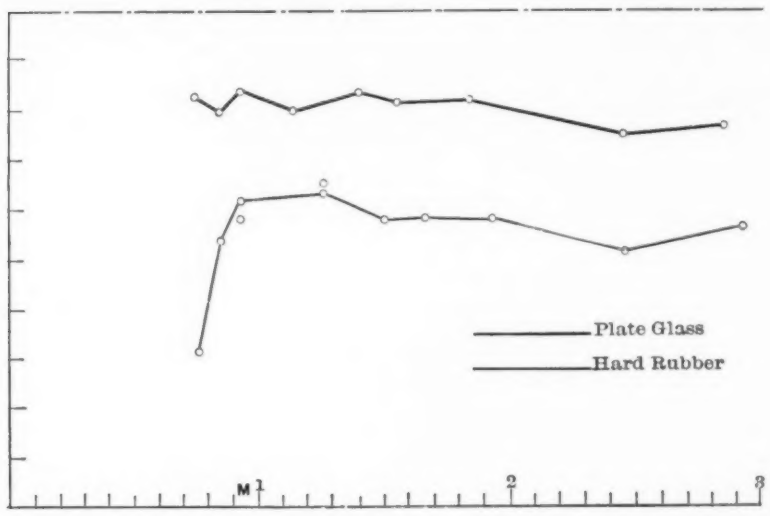


Fig. 2.

had been thinner or the glass thicker, the two spectra beyond this point might have been identical within the limits of probable error.

Figure 3 shows the same results in a slightly different way. The upper curve in this figure shows the distribution of energy in the normal spectrum of the glow-lamp used.[1] Wave-lengths are plotted as abscissæ, and corresponding intensities of radiant energy as ordinates. The distribution of energy in the prismatic spectrum was obtained by the same apparatus as that used in getting the transmission percentages, consequently the radiant energy before reaching the thermopile passed through two glass lenses, the glass sides of the prism, and the bisulphide of carbon. Because of the lack of selective absorption in glass in this region of the spectrum and the almost perfect diathermancy of carbon bisulphide

[1] Obtained by a method to be described in an article on Distribution of Energy in the Spectrum of the Glow-Lamp, by Dr. E. L. Nichols.

(Rubens,[1] Knut Ångstrom[2]), the only effect of this transmission is that all the ordinates of the curve in Fig. 3 are less than the true values by a constant percentage.

The distribution of energy in the normal spectrum is obtained from the distribution in the prismatic spectrum by correcting for the increasing condensation of rays in the direction of the longer wave-lengths.[3]

The ordinates of the middle curve are obtained by taking the product of the transmission percentage of glass for each wave-

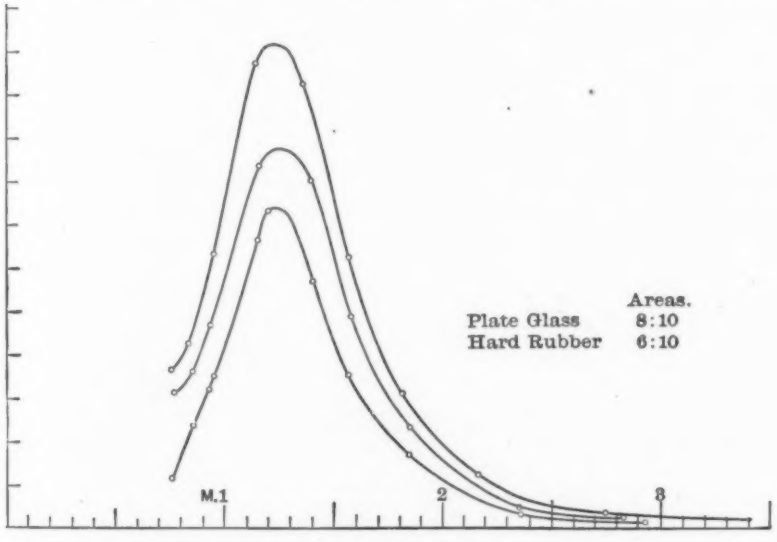


Fig. 3.

length and the ordinate of the energy curve of the lamp at that wave-length. The lower curve shows the distribution of energy in the transmission spectrum of hard rubber. From the way in which these curves are plotted the total infra-red energy transmitted by the substances varies as the areas included between the curves and the base line. In the case of glass and hard rubber the percentages of transmission for the different wave-lengths observed

[1] Annalen der Physik und Chemie, N. F. XLV., p. 260.

[2] Ofversigt af Kongl. Vetenskaps-Academiens Forhandlingar, 1892, No. 7, Stockholm, p. 345.

[3] Report on Mt. Whitney Expedition, p. 231; also Draper, American Journal of Science, 1872.

were so nearly constant that the observed points on the diagram were connected by a curve similar to the energy curve of the source.

Table V. contains the transmission percentages for quartz and lamp-black.

TABLE V.

λ	.778	.971	1.14	1.33	1.55	1.78	2.14
Quartz	81.2	88.7	90.0	91.9	90.9	92.8	94.5
Lamp-black . .	Too small to measure.	Too small to measure.	Too small to measure.	01.5	02.5	01.5	01.0

The quartz was an unusually clear plate of rock crystal 6.8 mm. thick, cut perpendicular to the optical axis of the crystal. The lamp-black was a deposit formed by smoking a plate of the glass described in Table IV. In this way a known medium was intro-

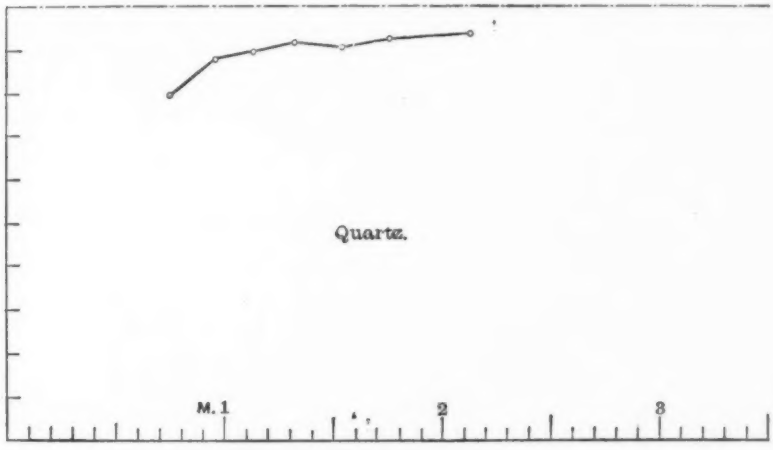


Fig. 4.

duced as a carrier for the smoke film. The film was thickened until a square centimeter of area was obtained. It was uniform and just heavy enough so that the outline of the filament of a glow-lamp at normal candle power was barely visible through it, when the lamp was held 30 cm. away from the eye in a dark room.

Figures 4 and 5 show these results plotted according to the conventions explained for Figs. 2 and 3. It will be noticed that the diathermancy of quartz increases with the wave-length as far,

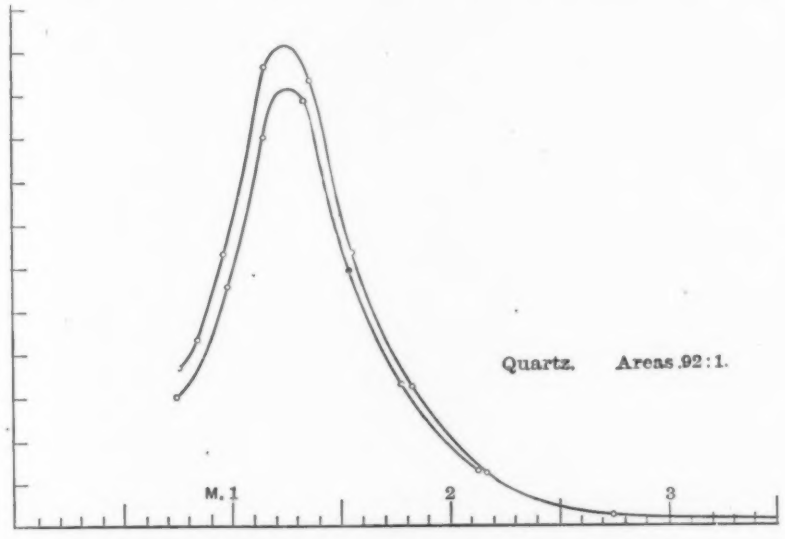


Fig. 5.

at least, as $\lambda = 2^{\mu}.14$. The results for lamp-black are not shown graphically.

Table VI. gives similar results for cobalt glass.

TABLE VI.

λ	.775	.862	.940	1.19	1.41	1.58	1.86	2.37	2.86
Cobalt glass. . .	79.6	80.1	66.7	20.0	20.7	22.6	45.7	55.4	52.9

The plate was 2.8 mm. thick. In the visible spectrum cobalt glass shows a number of absorption bands, and Table VI. shows the presence of a deep band in the infra-red beginning at $\lambda = 0^{\mu}.862$. At $\lambda = 1^{\mu}.86$ the diathermancy has increased somewhat from the lowest values, but it does not, within the region studied, reach its former value again. Figures 6 and 7 show the results for cobalt glass graphically.

Table VII. shows the transmission percentages for absolute alcohol and a solution of chlorophyll in alcohol. These percentages are for layers of alcohol and chlorophyll solution 1 cm. thick,

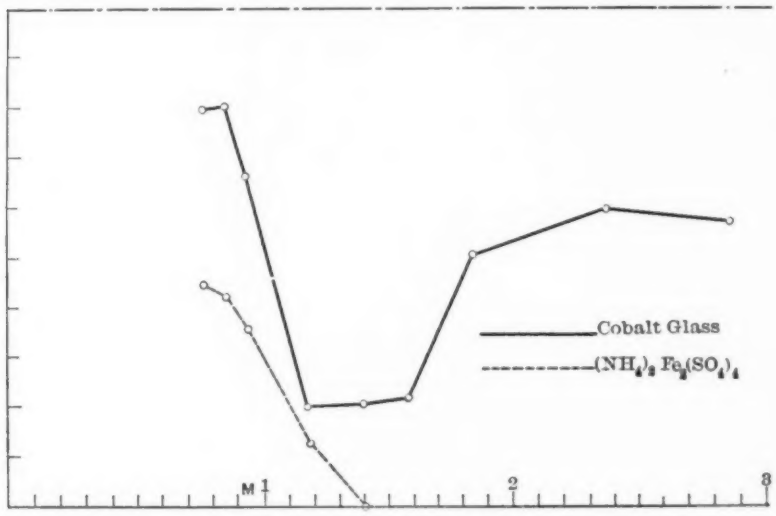


Fig. 6.

contained in a flat cell with thin glass walls. The percentages are computed on the basis of energy transmitted through glass sides and all. The same cell was used for both solutions, so the results are comparable.

TABLE VII.

λ	.776	.863	.941	1.19	1.41	1.58	1.85	2.32	2.83
Alcohol (C_2H_6O)	75.9	77.3	75.8	40.2	01.8	01.5	too small to measure	too small to measure	too small to measure
Chlorophyll . . .	66.1	77.1	70.3	32.1	01.9	too small to measure	"	"	"

At $\lambda = 0^{\mu}.863$ and $\lambda = 1^{\mu}.41$ the solutions were equal in absorbing power. The transmission spectra for both liquids came to an end near $\lambda = 1^{\mu}.4$. These results are shown graphically in Figs. 8 and 9.

Table VIII. contains the transmission percentage for the empty cell; for water; for a solution of oxyhæmoglobin, made up of 5 cc. cat's blood and 2cc. chloral in a liter of water; for a solution

of commercial potassium aluminium sulphate in water, saturated at 0°; and for a solution of commercial ammonium aluminium sulphate in water, saturated at 0°. Centimeter layers of all these

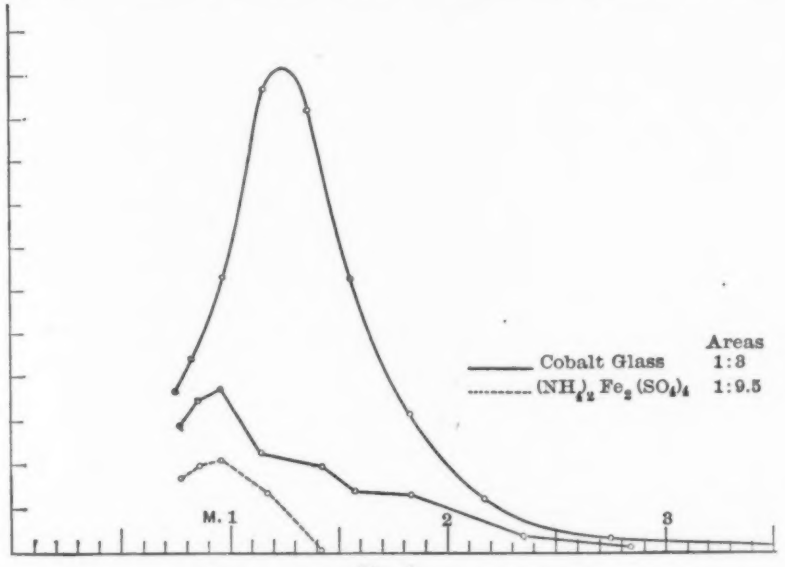


Fig. 7.

solutions were examined in the same cell, which was similar in every respect to the one used for alcohol and chlorophyll. These results are shown in Figs. 10 and 11.

TABLE VIII.

λ	.779	.865	.945	1.19	1.41	1.59	1.89	2.29	2.82
Empty cell . . .		72.	73.	73.					
Water	76.2	74.4	58.4	14.4	too small to measure	too small to measure	too small to measure	too small to measure	too small to measure
Blood	68.9	74.7	55.6	16.0	"	"	"	"	"
$K_2Al_2(SO_4)_4$ . .	73.6	72.2	51.3	16.2	"	"	"	"	"
$(NH_4)_2Al_2(SO_4)_4$	73.8	76.2	54.8	14.7	"	"	"	"	"

For $\lambda = 0^{\mu}.865$ the transmission of the empty cell is seen to be a trifle less than that of the cell filled with the several solutions. This is not an impossible relation, because the reflection from the inner surfaces of the cell is cut down considerably when the cell is

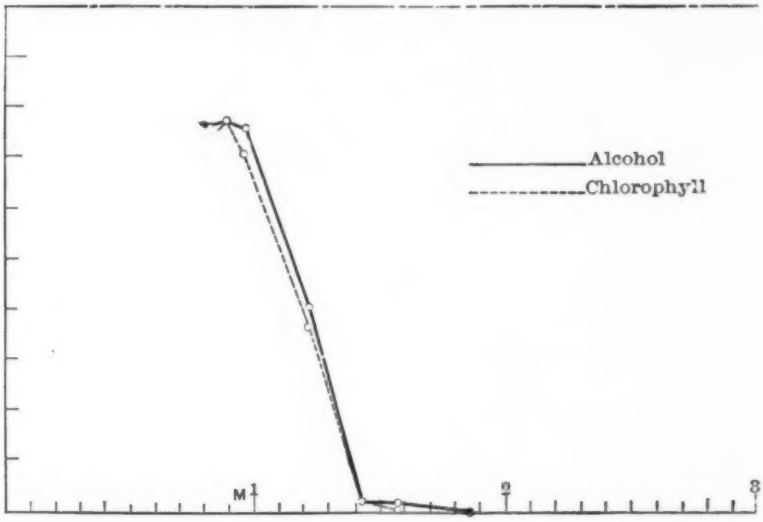


Fig. 8.

filled with the solutions, and the solutions were found highly diathermous at this point. The transmission spectra of these three solutions look very much like the spectrum of pure water. This seems to show that in alum solutions, commonly used to cut out dark heat, the alum is inactive. Whether alum in solution absorbs more radiant heat than distilled water has been questioned before, and H. N. Draper asks in "Nature,"[1] "Why use alum dissolved in water instead of water alone, to cut off obscure radiation?" In the next issue of that periodical he is answered on *a priori* grounds by Ch. Ed. Guillaume, that solid alum in its absorption occupies nearly the same place in the scale of solids that water does among liquids; consequently, if there be any difference in the selective absorption of the two substances, the alum solution will naturally be expected to cut out more radiation than the water alone.

[1] Vol. 44, p. 446.

Melloni,[1] using an Argand lamp with a glass chimney as a source, found that a layer of alum solution 9.21 mm. thick trans-

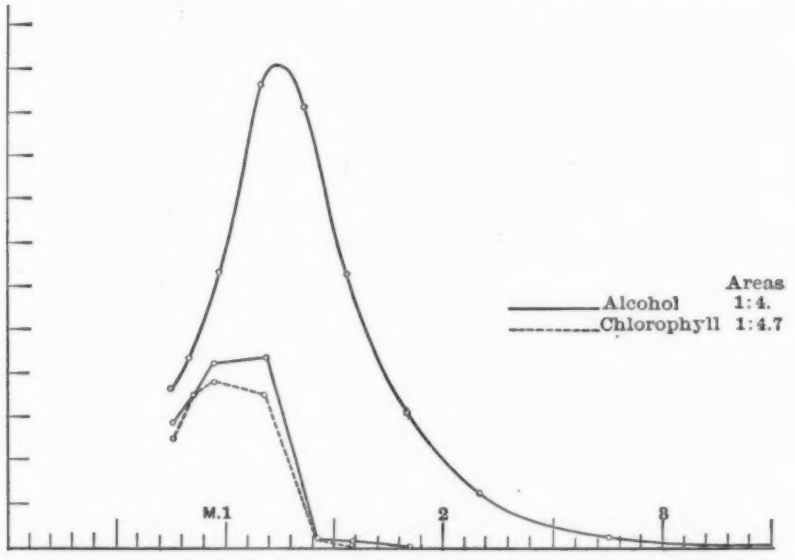


Fig. 9.

mitted 12 % of the total incident radiation, and that distilled water under like circumstances transmitted only 11 %.

Shelford Bidwell,[2] using a paraffine lamp as a light source and the thermopile, obtained the following results: —

Solutions.	Diathermancy.
Empty cell	1000
Water, distilled	197
Tap water	200
Alum, saturated solution	204

F. C. Porter,[3] using the electric arc at 40 volts and 6.4 ampères, as a source and the Crooks radiometer, reached the following results: —

[1] Thermochrose, p. 165. [2] Nature, Vol. 44, p. 565. [3] Nature, Vol. 45, p. 29.

	No. of revolutions of radiometer arms.	Time in half-seconds.
1. Unimpeded radiation	25	160
2. Through empty cell	25	254
3. Unimpeded radiation	25	152
4. Cell + water	25	820
5. Unimpeded radiation	25	164
6. Cell + water (second experiment) . .	25	832
7. Unimpeded radiation	25	160
8. Cell + alum solution	25 (time of two revolutions actually taken)	43,200
9. Unimpeded radiation	25	160

The spectra of all these solutions stopped before wave-length 1$^{\mu}$.41 was reached. It was thought that possibly the quantity of alum in a centimeter layer of the solutions was not enough to pro-

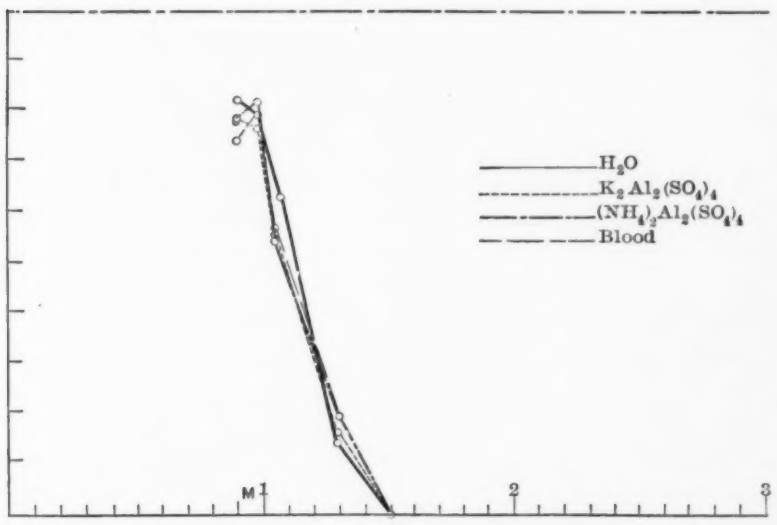


Fig. 10.

duce a noticeable effect ; consequently layers two centimeters thick were examined under like circumstances. The results appear in Table IX.

Fig. 1.

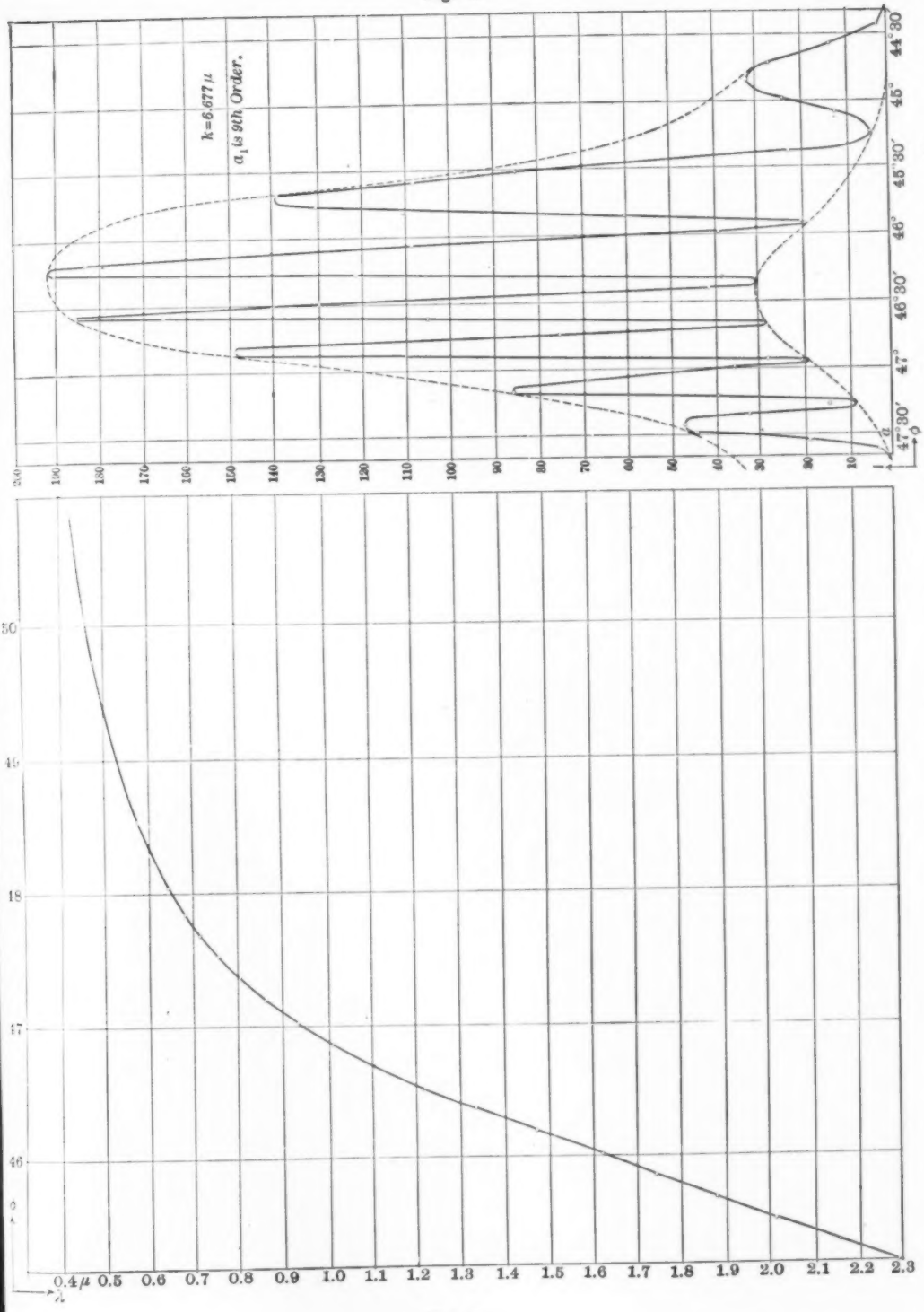
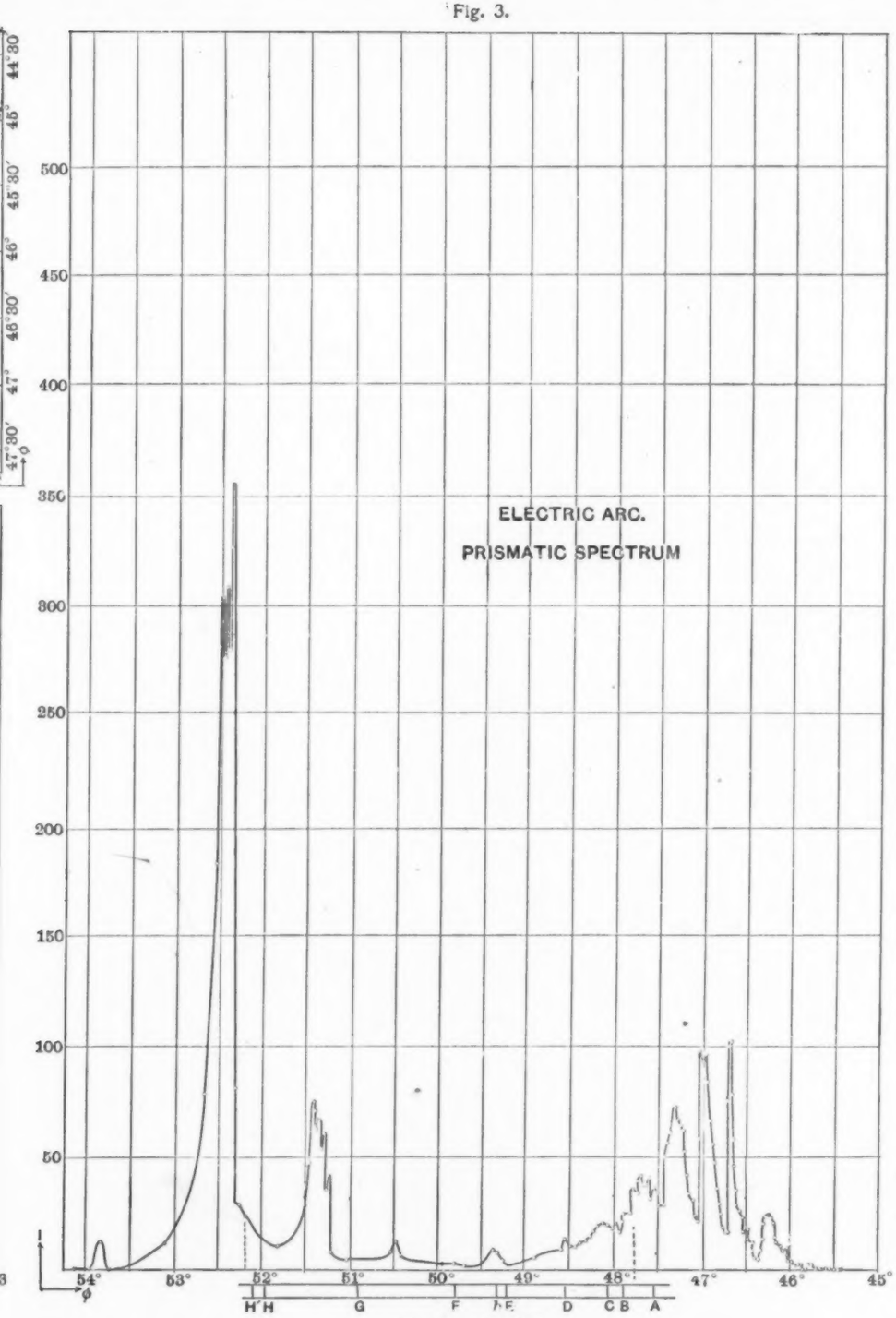

λ=6.677 μ
a₁ is 9th Order.
44°30′
45°
45°30′
46°
46°30′
47°
47°30′
φ
240 230 220 210 200 190 180 170 160 150 140 130 120 110 100 90 80 70 60 50 40 30 20 10
50
49
18
17
46
0.4μ 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3
λ

Fig. 2.

Fig. 3.



B. W. SNOW.

Fig. 4

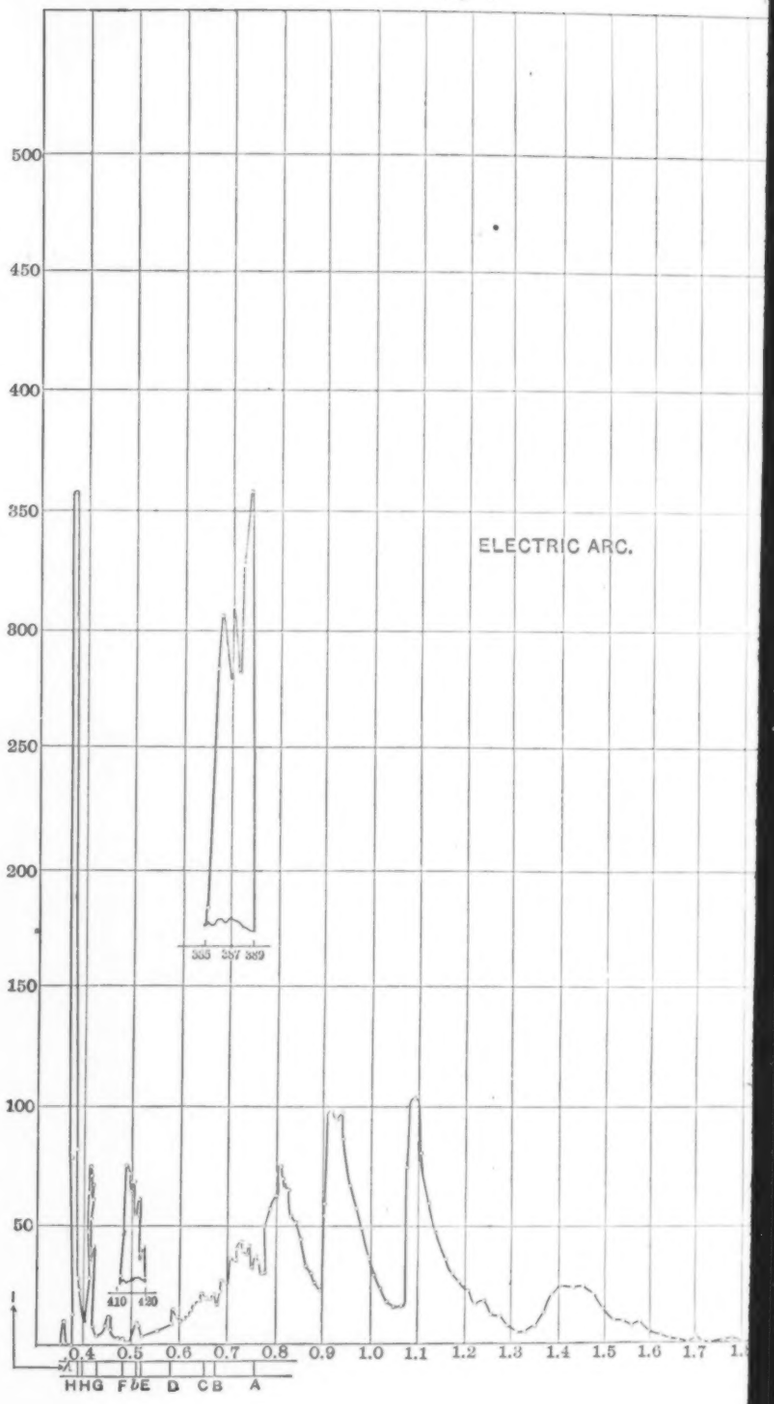
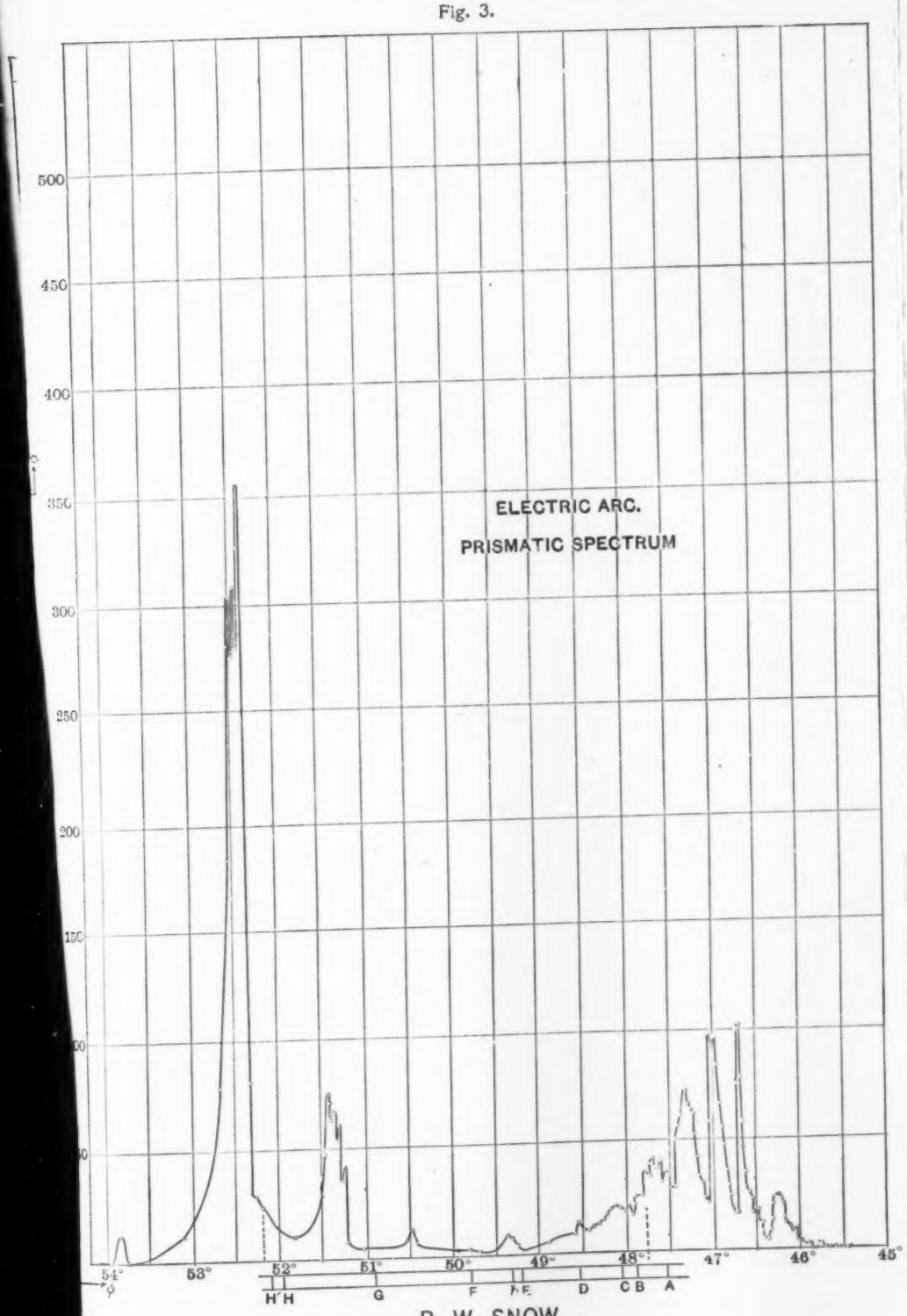


Fig. 3.


500
450
400
350
300
250
200
150
100
50
ELECTRIC ARC.
PRISMATIC SPECTRUM
54°
53°
52°
51°
50°
49°
48°
47°
46°
45°
H' H
G
F
E
D
C B
A

B. W. SNOW.

Fig. 4 PLATE I.

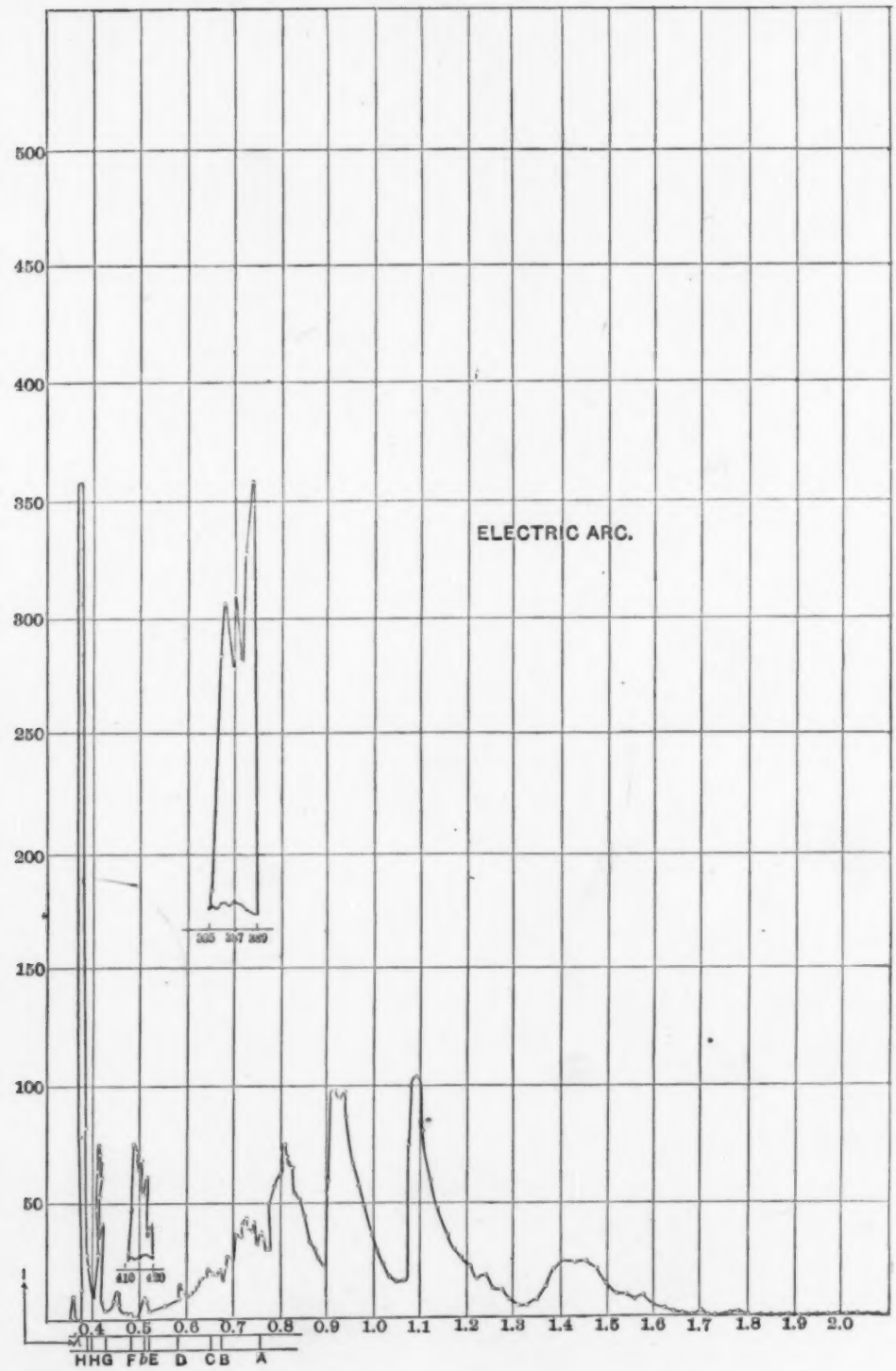

ELECTRIC ARC.
500
450
400
350
300
250
200
150
100
50
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
HHG
FDE
D
CB
A

TABLE IX.

λ	.779	.865	.945	1.19	1.41	1.59	1.89	2.29	2.82
Water	77.1	71.7	44.2	03.7	too small to measure	too small to measure	too small to measure	too small to measure	too small to measure
Blood	72.0	68.4	42.3	05.1	"	"	"	"	"
$K_2Al_2(SO_4)_4$ . .	74.1	69.5	39.5	03.9	"	"	"	"	"
$(NH_4)_2Al_2(SO_4)_4$	75.8	69.6	42.6	05.4	"	"	"	"	"

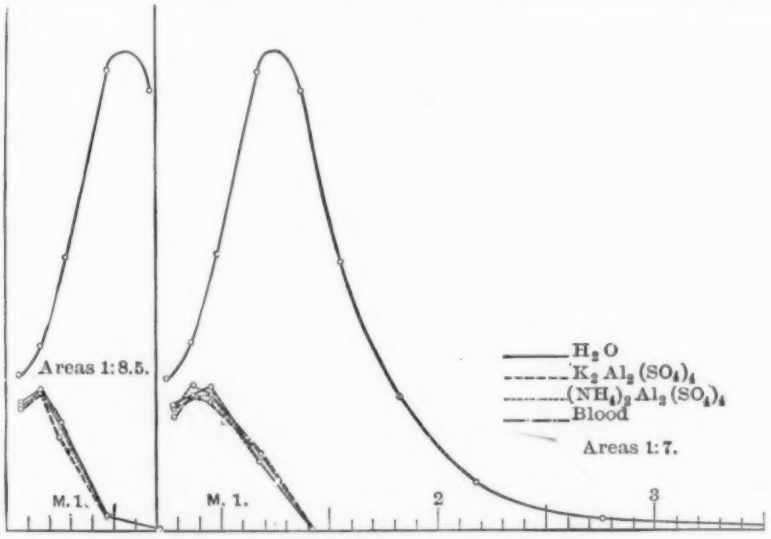


Fig. 11.

Table X. gives the transmission percentages for a centimeter layer of water solution of commercial ammonium-iron alum saturated at 0°. The cell was the one used for the other alum solutions.

TABLE X.

λ	.777	.863	.943	1.19	1.41	1.58	1.88	2.30	2.80
$(NH_4)_2Fe_2(SO_4)_4$	45.0	42.9	35.5	12.5	too small to measure	too small to measure	too small to measure	too small to measure	too small to measure

The contents of this table are shown graphically in Figs. 6 and 7.

Summary of Results.

1. The diathermancy of plate glass is fairly uniform through the infra-red down to $\lambda = 3$. The highest and lowest percentages of transmission are 82 and 72.

2. The diathermancy of hard rubber rises very rapidly just beyond the limits of the visible spectrum reaching the percentage 59 of transmission at $\lambda = 0^{\mu}.940$. It does not rise above 65% nor fall below 52% between $\lambda = 0^{\mu}.940$ and $\lambda = 3^{\mu}$.

3. The diathermancy of quartz increases with the wave-length from the limit of the visible spectrum to $\lambda = 3^{\mu}$.

4. Lamp-black absorbs almost non-selectively throughout the region studied.

5. Cobalt glass has a wide absorption band between $\lambda = 0^{\mu}.862$ and $\lambda = 1^{\mu}.85$.

6. The diathermancy of alcohol is nearly constant between $\lambda = 0^{\mu}.776$ and $\lambda = 0^{\mu}.941$. Beyond the latter wave-length it decreases rapidly and ceases at $\lambda = 1^{\mu}.4$. Chlorophyll in solution does not decrease the diathermancy of alcohol at $\lambda = .863^{\mu}$ and at $\lambda = 1^{\mu}.41$.

7. Neither potassium alum nor ammonium alum in solution alters the diathermancy of distilled water in the region studied.

8. Ammonium-iron sulphate in solution decreases the diathermancy of distilled water.

The work described in the foregoing paper was done in the Physical Laboratory of Cornell University during July and August, 1892, under the direction of Dr. Edw. L. Nichols, to whom I take pleasure in acknowledging my indebtedness for assistance and many important suggestions. I am indebted to Mr. E. S. Ferry also for help in making some of the observations preliminary to this study.

COLGATE UNIVERSITY, HAMILTON, N.Y.
January, 1893.

ON THE RELATION BETWEEN THE LENGTHS OF THE YARD AND THE METER.

By William A. Rogers.

THE present determination of the length of the meter, expressed in terms of the yard, rests upon a combined yard and meter constructed by the writer in 1880. The description of the method of construction employed, and of the authorities upon which the total lengths depend, will be found in the Proceedings of the American Academy of Arts and Sciences for 1882–83, Vol. XVIII. It will be useful to give here a brief recapitulation of the results there found.

(*a*) Diagram showing the form of the bronze bar R_2, the ingredients of which are 16 parts of copper, $2\frac{1}{2}$ parts of tin, and 1 part of zinc.

The inner circles, *a*, *b*, *c*, represent the polished surfaces of platinum-iridium plugs inserted in the metal. The lines traced

upon these plugs are protected by circles of thin cover-glass cemented to the surface of the metal by means of Canada balsam. These cover-glasses are represented by the larger circles.

(*b*) Corrections to the yard (*ab*) and the meter (*ac*) adopted in 1882.

(*ab*) + .000008 in. = the Imperial Yard of Great Britain = Y;
(*ac*) − 1.3μ = the Metre des Archives = A_0.

Coefficient of expansion of the bronze bar $R_2 = 17.17\mu$ for each degree C.

(*c*) The meter depends on the following authorities:—

(1) A meter upon a copper bar having the X form, presented to the writer by the late Professor Tresca, who was at that time in charge of the Conservatoire des Arts et Metiers at Paris. This meter, designated *T*, was transferred by M. Tresca from a copy of the Metre des Archives, designated "No. 19," which was at that time provisionally adopted as the equivalent of the original standard. The transfer was made at two o'clock in the morning of February 6, 1880. From subsequent comparisons it appeared that —

$$T - \quad 1.0\mu = \text{"No. 19" at } 13.70^\circ \text{ C.},$$

and

$$T - 118.9\mu = \text{Metre des Archives at } 0^\circ \text{ C.}$$

(2) An end-measure meter by Foment = *F*, of which the following relation to A_0 had been found:—

$$F - 8.4\mu = A_0 \text{ at } 0^\circ \text{ C.}$$

(3) A combined yard and meter upon brass, made at the Bureau of Weights and Measures, Washington, and now in the possession of the Stevens Institute. The meter upon this bar, which is designated *CS*, was in 1883 compared with the provisional standard at that time adopted at the International Bureau of Weights and Measures with the resulting relation:—

$$CS + 310\mu = A_0 \text{ at } 0^\circ \text{ C.}$$

(4) A meter belonging to the United States Coast Survey, known as the Berlin meter "No. 49," for which the following relation is given by Foester:—

$$\text{"No. 49"} + 21.4\mu = A_0 \text{ at } 0^\circ \text{ C.}$$

The yard upon R_2 depends upon the following authorities:—

(1) A yard R_n upon nickel-plated steel made by the writer in 1880, and compared with *Y* by Mr. H. J. Chaney, the Warden of the Imperial Standards.

$$R_n + .000316 \text{ in.} = Y \text{ at } 62^\circ.$$

(2) A yard upon the brass bar *CS*, also compared with *Y* by Mr. Chaney.

$$CS + .000081 \text{ in.} = Y \text{ at } 62^\circ.$$

(3) The yard of the United States Bureau of Weights and Measures known as "Bronze 11."

$$\text{"Bronze 11"} + .000088 \text{ in.} = Y \text{ at } 62°.$$

Two values of the relation between the relative lengths of the yard and the meter have been published by the writer. The first one is given on page 398 of the Proceedings of the Academy, viz.: —

$$Y + 3.37027 \text{ in.} = A_0 \text{ at } 62°.$$

During the year 1887 the writer constructed for the Standards' Department of the London Board of Trade a combined yard and meter upon a bar of Baily's metal, which was transmitted by Mr. Chaney for this purpose. Before this standard, designated C_2, was returned to London it was compared with R_2, with the following results: —

For the Meter.			For the Yard.
No. of Comparisons.	τ	$R_2 - C_2$	$R_2 - C_2$
8	31.66°	-2.7μ	-0.1μ
9	33.83	-2.3	-1.2
9	40.42	-1.7	-0.2
8	47.52	-1.7	$+0.8$
8	72.75	-2.9	-0.8
	Means	-2.26	-0.30

Soon after the arrival of the standard at London a series of comparisons with the Imperial yard was made by Mr. Chaney. The results are given on page 7 of the Report to the Board of Trade for 1886–88, from which it appears that —

$$C_2 + .000128 \text{ in.} = Y.$$

Since $C_2 - .000012 = R_2$, we have $R_2 + .000140$ in. $= Y$.

The original relation given must therefore be corrected by $+.000132$ in. and the relation between the length of the yard and the meter is —

$$Y + 3.37014 \text{ in.} = A_0.$$

A remeasurement of the distance (*bc*) made at this time, the details of which are omitted, gave, using the new correction to R_2, —

$$Y + 3.37012 \text{ in.} = A_0.$$

This value was communicated to Professor Hazen of the Signal Office at Washington and was made the basis of the linear measure tables in his Handbook of Meteorological Tables, published in 1888. In the letter in which this result was communicated, the writer somewhat incautiously expressed the opinion that the true relation would be found to fall between the limits 39.37012 in. and 39.37015 in. In the preface to the volume, the first value is erroneously printed 39.3702 in. With the limitation that the corrections to the yard and the meter of R_2 are correctly given, this statement will probably be found to hold nearly true.

In view of the importance of a direct measurement of the space (*bc*) without the interference of the cover-glasses, it was decided to transfer to the edge of R_2 a new yard and meter, and in order that the conditions of comparison might vary as widely as possible, a provisional yard and meter were first laid off with the following subdivisions: —

Meter designated $R_{2_1} = (ac)$. Yard designated R_{2_1} was subdivided into 9 equal parts. Two separate 4-inch spaces were subdivided to inches, and the last inch to tenths of inches. The space from 3.3 in. to 3.4 in. was further subdivided into 10 equal parts, thus allowing a direct comparison of the space (*bc*) with the nearest hundredth of an inch. These auxiliary scales are designated $4\,i_1$ and $4\,i_2$. After the entire investigation had been completed, a second transfer was made to the other edge of the bar in which the 4-inch subdivisions of the yard were given rather large periodic errors. A new 4-inch space was also laid off. The former is designated R_{2_2} and the latter $4\,i_3$.

The determination of the true value of (*bc*) consists of three operations, viz.: —

(*a*) The measure of the true value of the space $4\,i$ obtained by comparing it with each of the 4-inch spaces of the yard, corrected for the amount of the error in the total length. The correction for the line 3.37 in. will thus become known.

(*b*) The comparison of the space (*bc*) with the $4i$ space, the deviation from the distance 3.37 in. being measured with the micrometer of the microscope.

(*c*) The subtraction from the measured value of (*bc*) of the algebraic sum of the corrections to the distances (*ac*) and (*ab*). Thus, since the meter of R_2 is 1.3μ too long, and the yard is .00014 in. too short, the space (*bc*) will be .00019 in. too long.

The results, expressed in divisions of the micrometer, of the different measurements of the space (*bc*), are given below. One division of micrometer $= .203\mu = .000008$ in.

I. Measurement of (*bc*) in terms of a 4-inch scale upon speculum metal. This scale has the last inch subdivided into 100 equal parts.

II. Measurement of (*bc*) in terms of a decimeter subdivided to millimeters, with the last millimeter subdivided to tenths of millimeters. For a description of these scales, see Astronomical Journal, No. 281.

Date.	T	I. Excess over 3.37 in. at 62°.	II. Excess over 85.6 mm. at 62°.
1893 Jan. 14	40.3°	− 49.3 div.	− 37.7 div.
" 15	49.5	− 45.0	− 35.7
" 15	56.3	− 48.6	− 34.1
" 16	57.8	− 48.4	− 34.1
" 17	50.6	− 44.6	− 39.1
" 18	61.8	− 45.0	− 33.8
" 19	62.0	− 46.2	− 33.3
" 19	68.6	− 48.4	− 34.4
" 20	50.8	− 48.6	− 34.1
" 21	46.4	− 43.2	− 35.9
" 22	36.0	− 45.7	− 33.6
		means − 46.6	− 35.1

I. Measured excess over 3.37 in. . . . . − 46.6 div.
Space (*bc*) too long . . . . . + 24.0
Correction to line 3.37 in. . . . . + 3.5
Excess over 3.37 in. . . . . − 19.1
(*bc*) = 3.370153 in.

II. Measured excess over 85.600 mm. . . . − 35.1 div.
Space (*bc*) too long . . . . . + 24.0
Correction to line 85.6 mm. . . . . + 1.8
Excess over 85.6 mm. . . . . − 9.3
(*bc*) = 85.60186 mm. = 3.370154 in.

III. Measurement of (bc) of R_2 and of (bc) of $R_{2\varepsilon_1}$ in terms of $4i_1$ and $4i_2$.

RELATIONS FOR TOTAL LENGTH.

For meter R_2,	*For yard* R_2,	*For meter* $R_{2\varepsilon_1}$,	*For yard* $R_{2\varepsilon_1}$,
$R_2 - 6.5$ div. $= A_0$.	$R_2 + 17.5$ div. $= Y$.	$R_{2\varepsilon_1} + 1.2$ div. $= A_0$.	$R_{2\varepsilon_1} + 13.6$ div. $= Y$.

Cumulative corrections to the 4-inch subdivisions of the yard $R_{2\varepsilon_1}$: —

Subdivisions in inches	4	8	12	16	20	24	28	32	36
Corrections	$+1.0^d$	-2.9^d	-1.5^d	$+4.8^d$	$+6.7^d$	$+4.1^d$	$+5.6^d$	$+2.8^d$	$+0.0^d$.

CORRECTIONS TO SPACES $4i_1$ AND $4i_2$.

In the following tables the first column represents the summed series of corrections to the separate inches which are marked [ ], and the distributed corrections for each tenth of the fourth inch. The second column contains the corrections to the 4-inch space due to the error in total length. The third column contains

TABULAR CORRECTIONS TO $4i_1$.									TABULAR CORRECTIONS TO $4i_2$.							
Lines.	Σ.			Sum.	$\frac{1}{100}$ in.				Σ.				$\frac{1}{100}$ in.			
in.	div.	div.	$\frac{1}{10}$ in.	div.					div.	div.	$\frac{1}{10}$ in.	div.				
0	+ 0	+0		+ 0					+0	+ 0		+ 0				
[1]	+23	+0		+23					+9	− 9		+ 0				
[2]	+37	+0		+37	in.	div.	div.	div.	+6	−18		−12	div.	div.	div.	
[3]	+45	+0	+ 0	+45	3.30	+46	+0	+46	+9	−27	+ 0	−18	−24	+ 0	−24	
3.1	+40	+1	+ 8	+49	3.31	+43	+0	+43	+8	−28	+10	−10	−24	− 1	−25	
3.2	+36	+1	+20	+57	3.32	+39	−3	+36	+7	−29	+14	− 8	−24	− 9	−33	
3.3	+31	+1	+14	+46	3.33	+36	−9	+27	+6	−30	+ 0	−24	−24	−15	−39	
3.4	+27	+1	−16	+12	3.34	+32	−7	+25	+5	−31	+ 0	−26	−24	−15	−39	
3.5	+22	+1	−10	+13	3.35	+29	−7	+22	+4	−32	+ 2	−26	−24	−11	−35	
3.6	+18	+1	− 8	+11	3.36	+25	−2	+23	+3	−33	+ 5	−25	−25	− 3	−28	
3.7	+13	+2	− 1	+14	3.37	+22	−3	+19	+2	−34	+ 6	−26	−25	− 4	−29	
3.8	+ 9	+2	+10	+21	3.38	+18	−2	+16	+2	−35	+16	−17	−25	− 8	−33	
3.9	+ 4	+2	+11	+17	3.39	+15	−2	+13	+1	−36	+10	−25	−26	+ 0	−26	
[4.0]	+ 0	+2	+ 0	+ 2	3.40	+12	+0	+12	+0	−37	+ 0	−37	−26	+ 0	−29	

SPACES (*bc*) IN TERMS OF THE CORRECTED VALUES OF $4i_1$ AND $4i_2$.

Date.	*From* R_2.	*From* R_{2e_1}.	*From* R_2.	*From* R_{2e_1}.
Jan. 30.	−58.8 div.	−12.2 div.	−49.5 div.	− 2.9 div.
" 31.	−66.8	−15.1	−60.3	− 6.4
Feb. 1.	−64.1	−11.7	−55.6	− 3.2
" 1.	−65.4	−15.2	−56.0	− 5.8
" 1.	−60.2	−12.5	−54.9	− 4.8
" 2.	−59.4	− 9.8	−49.0	+ 0.6
Means.	−62.4	−12.7	−54.2	− 3.7
Corrections to (*bc*). . . .	+24.0	+24.0	+12.4	+12.4
Corrections to line 3.37 in.	+19.0	−29.0	+19.0	−29.0
Excess over 3.37 in. . . .	−19.4	−17.7	−22.8	−20.3
Excess in parts of an inch.	−.000155	−.000142	−.000182	−.000162

the corrections to the $\frac{1}{10}$-inch subdivisions of the fourth inch. The fourth column is formed by taking the sum of the three previous columns. The subsidiary tables contain the corrections to the lines from 3.30 in. to 3.40 in. The correction to line 3.37 in. is required.

IV. Measurement of (*bc*) of R_2 and of R_{2e_2} in terms of $4i_2$.

In order to introduce a wide variation in the conditions of comparison, the corrections are given for both edges of the terminal lines of the yard and the meter R_{2e_2}.

RELATIONS FOR TOTAL LENGTH.

For the right edge of the terminal line of —		For the left edge of the terminal line of —	
The meter.	*The yard.*	*The meter.*	*The yard.*
R_{2e_2} − 6.2 div. = A_0.	R_{2e_2} + 0.1 div. = Y.	R_{2e_2} − 49.5 div. = A_0.	R_{2e_2} − 37.0 div. = Y.

Cumulative corrections to the 4-inch divisions of the yard R_{2e_2}: —

Subdivisions in inches	4	8	12	16	20	24	28	32	36
Corrections	−9.3^d	−15.0^d	−16.6^d	−12.9^d	−22.0^d	−18.0^d	−5.3^d	−4.9^d	+0

CORRECTIONS TO THE 4-INCH SPACE $4\,i_3$.

Comparison with the Right Edge of the Terminal Line of the $4\,i_3$ Space.									Comparison with the Left Edge of the Terminal Line of the $4\,i_3$ Space.							
Lines.	Σ.			Sum.	$\frac{1}{100}$ inch.						Sum.		$\frac{1}{100}$ inch.			
in.	div.	div.	$\frac{1}{10}$ inch.		in.	div.	div.	div.	div.	div.	$\frac{1}{10}$ inch.	div.	in.	div.	div.	div.
0	+ 0.0	0.0		0.0					0.0	0.0						
[1]	− 0.6	+ 0.5		− 0.1					+ 9.6	− 10.1		− 0.5				
[2]	+ 5.9	+ 1.1		+ 7.0					+ 26.2	− 20.2		+ 6.0				
[3.0]	+ 2.7	+ 1.6	+ 0.0	+ 4.3	3.30	+ 4.1	+ 0.0	+ 4.1	+ 33.0	− 30.3	+ 0.0	+ 2.7	3.30	+ 2.7	+ 0.0	+ 2.7
3.1	+ 2.4	+ 1.7	− 2.9	+ 1.2	3.31	+ 3.8	+ 3.3	+ 7.1	+ 29.7	− 31.3	+ 1.2	− 0.4	3.31	+ 2.4	+ 3.3	+ 5.7
3.2	+ 2.1	+ 1.7	+ 4.6	+ 8.4	3.32	+ 3.5	− 4.1	− 0.6	+ 26.4	− 32.3	+ 12.8	+ 6.9	3.32	+ 2.1	− 4.1	− 2.0
3.3	+ 1.9	+ 1.8	+ 0.4	+ 4.1	3.33	+ 3.3	+ 3.0	+ 6.3	+ 23.1	− 33.3	+ 12.7	+ 2.5	3.33	+ 1.8	+ 3.0	+ 4.8
3.4	+ 1.6	+ 1.8	− 2.0	+ 1.4	3.34	+ 3.0	− 3.9	− 0.9	+ 19.8	− 34.4	+ 14.4	− 0.2	3.34	+ 1.5	− 3.9	− 2.4
3.5	+ 1.3	+ 1.9	− 5.2	− 2.0	3.35	+ 2.7	− 3.9	− 1.2	+ 16.5	− 35.4	+ 15.3	− 3.6	3.35	+ 1.2	− 3.9	− 2.7
3.6	+ 1.1	+ 1.9	− 8.9	− 5.9	3.36	+ 2.5	+ 1.8	+ 4.3	+ 13.2	− 36.4	+ 15.7	− 7.5	3.36	+ 0.9	+ 1.8	+ 2.7
3.7	+ 0.8	+ 2.0	− 4.8	− 2.0	3.37	+ 2.2	− 0.2	+ 2.0	+ 9.9	− 37.5	+ 23.9	− 3.7	3.37	+ 0.5	− 0.2	+ 0.3
3.8	+ 0.5	+ 2.0	− 2.4	+ 0.1	3.38	+ 1.9	− 4.1	− 2.2	+ 6.6	− 38.5	+ 30.4	− 1.5	3.38	+ 0.2	− 4.1	− 3.9
3.9	+ 0.3	+ 2.1	+ 0.2	+ 2.6	3.39	+ 1.6	+ 1.1	+ 2.7	+ 3.3	− 39.5	+ 37.1	+ 0.9	3.39	− 0.1	+ 1.1	+ 1.0
4.0	+ 0.0	+ 2.2	+ 0.0	+ 2.2	3.40	+ 1.4	+ 0.0	+ 1.4	+ 0.0	− 40.5	+ 0.0	− 40.5	3.40	− 0.4	+ 0.0	− 0.4

The mean of the two values + 2.0 and + 0.3 of the correction for line 3.37 in., viz. + 1.1 div., will be adopted.

SPACES (bc) IN TERMS OF THE CORRECTED VALUE OF $4\,i_3$.

Date.	$R_3 - 4\,i_3$.	$R_{2_2} - 4\,i_3$.	$R_2 - 4\,i_3$. Left edge.	$R_{2_3} - 4\,i_3$. Right edge of line for meter. Left edge of line for yard.	$R_{2_4} - 4\,i_3$. Left edge of line for meter. Right edge of line for yard.
1893. Feb. 27	−49.9 div.	−27.0 div.	−33.9 div.	+10.7 div.	−70.1 div.
" 28	−48.0	−30.9	−32.8	+11.0	−68.8
" 28	−45.6	−29.4	−33.0	+14.0	−67.5
" 28	−44.0	−28.8	−37.2	+16.8	−72.2
" 29	−44.2	−24.9	−36.6	+16.2	−63.0
" 29	−48.2	−24.0	−34.9	+13.8	−67.9
" 29	−45.1	−24.9	−29.7	+11.5	−65.7
Mar. 1	−47.6	−26.7	−32.4	+11.8	−69.1
" 1	−43.4	−24.8	−31.9	+17.3	−70.0
Means.	−46.2	−26.8	−33.6	+13.7	−68.3
Corrections to (bc). . .	+24.0	+ 6.3	+12.5	−30.8	+49.6
Correction of line 3.37 in.	+ 1.1	+ 1.1	+ 1.1	+ 1.1	+ 1.1
Excess over 3.37 in. .	−21.1	−19.4	−20.0	−16.0	−17.6
Excess in parts of an inch.	−.000169	−.000155	−.000160	−.000128	−.000141

Adopting the mean of the 11 determinations, we have finally: —

$$Y + 3.370155 \text{ in.} = A_0.$$

Dr. Gould, in his annual address before the American Meteorological Society, gives for the result of the latest determination at Breteuil: —

$$Y + 3.37000 \text{ in.} = A_0.$$

It is to be noted, however, that this determination is the result of only secondary comparisons through the toise. If this relation should eventually be found to be true, meter R_2 would require a further correction of -3.9μ, on the supposition that the correction for the yard is correctly given.

SHANNON PHYSICAL LABORATORY, COLBY UNIVERSITY,
March 29, 1893.

ON THE INFRA-RED SPECTRA OF THE ALKALIES.[1]

BY BENJAMIN W. SNOW.

I. INTRODUCTION.

OUR knowledge of the infra-red portion of the solar spectrum is confined almost entirely to investigations which have been made during the last fifty years. As early as the year 1840, Sir John Herschel[2] succeeded in showing an uneven distribution of energy in that part of the solar spectrum lying beyond the red, by allowing the spectrum of the sun's rays to fall upon a paper moistened with alcohol, and observing that at certain places in the invisible spectrum the paper dried more rapidly than at others. From this he concluded that the continuous solar spectrum is crossed at intervals by cold bands.

Later investigators have employed a variety of methods to confirm these results. Draper,[3] the two Becquerels[4] and Lommel[5] by means of phosphorescent plates, Fizeau and Foucault[6] by using a thermometer, Lamansky,[7] Mouton,[8] and Dessains[9] with the thermopile, Abney[10] by photographic methods, and finally

[1] From Wiedemann's Annalen Bd. 47 (communicated by the author).

[2] *Sir John Herschel*, Proc. Roy. Soc., p. 209, 1840. Phil. Mag., **16**, p. 331, 1840.

[3] *Draper*, Phil. Mag., **24**, p. 456, 1842; **11**, p. 157, 1881.

[4] *E. Becquerel*, Compt. rend., **69**, p. 999, 1869; **77**, p. 302, 1873. *H. Becquerel*, Compt. rend., **96**, p. 123, 1883; **99**, p. 417, 1884. Ann. de Chim. et de Phys., **30**, p. 33, 1883.

[5] *Lommel*, Wied. Ann., **40**, p. 681, 1890; **40**, p. 687, 1890.

[6] *Fizeau* and *Foucault*, Compt. rend., **25**, p. 449, 1847.

[7] *Lamansky*, Monatsber. der k. Acad. d. Wiss. zu Berlin, p. 638, 1871. Pogg. Ann., **146**, p. 207, 1872. Phil. Mag., **43**, p. 282, 1872.

[8] *Mouton*, Compt. rend., **89**, p. 298, 1879.

[9] *Dessains*, Compt. rend., **95**, p. 435, 1882.

[10] *Abney*, Phil. Trans., II., p. 653, 1880; p. 457, 1886.

Langley[1] with the aid of the bolometer, have not only verified the three bands first suspected by Herschel, but have found many new ones, and have given to each its respective wave-length.

Captain Abney appears to have been the first person who succeeded in studying the infra-red spectra of the metals. In a paper[2] published in 1879, he stated that the emissive spectrum of lithium contains an infra-red line which coincides in position with a line previously discovered by him in the absorption spectrum of this metal. Later experiments[3] showed that sodium also, when burned in the electric arc, has two infra-red lines having wave-lengths $\lambda = .8187\,\mu$ and $\lambda = .8199\,\mu$ respectively. The intensity of these lines was estimated at 3, the intensity of the D lines being taken as 10.

Apparently without a knowledge of Abney's work, H. Becquerel found, during the next four years, that many other metals besides sodium and lithium have interesting spectra in the infra-red. The two sodium lines discovered by Abney were again determined by Becquerel, and a third line of still greater wave-length observed.

His first method[4] for determining the wave-lengths of these lines consisted in projecting upon the slit of the spectrometer a real image of the arc in which the metals were burned, and receiving upon a phosphorescent plate the spectrum produced by a calibrated prism. These experiments were repeated a year later,[5] when the prism producing the spectra was replaced by a Rutherford grating. Mention will again be made of these researches of Becquerel, and a comparison drawn between his results and those now under consideration, when the latter are discussed at the close of the present paper.

From conclusions reached in a recent series of investigations, Kayser and Runge[6] have shown, in an article "Ueber die Spectren

[1] *Langley*, Report of the Mt. Whitney Expedition, p. 131, 1884. Phil. Mag. (5), 26, p. 511, 1888.

[2] *Abney*, Phil. Mag. (5), 7, p. 316, 1879.

[3] *Abney*, Proc. Roy. Soc., 32, p. 443, 1881.

[4] *H. Becquerel*, Compt. rend., 97, p. 71, 1883. Ann. de Chim. et de Phys., 30, p. 45, 1883.

[5] *H. Becquerel*, Compt. rend., 99, p. 374, 1884.

[6] *Kayser* und *Runge*, Wied. Ann., 41, p. 306, 1890.

der Alkalien," that the wave-lengths of the spectral lines of the metals belonging to this group bear to each other a mathematical relation, and may be represented by a formula. The constants of these formulæ, being determined by measurements made in the visible and ultra-violet portions of the spectrum, can rightfully be used, strictly speaking, only in those regions for which they were found. Since, however, these authors ascribe to their formulæ a general validity, and inasmuch as the wave-lengths of the infra-red lines obtained from them by extrapolation do not in all cases coincide with the results of previous observers, it appears to the present writer not altogether without interest to submit anew to a careful study the heat spectrum of the alkaline metals.

II. EXPERIMENTAL METHODS AND APPARATUS.

1. *The Bolometer.*

For a considerable period the bolometer has proven itself to be the best means of studying spectra in the infra-red. This instrument in the perfected state, to which it has been developed by such men as Langley, Ångström, Robert von Helmholtz, and Rubens, fulfilled the conditions necessary to the solution of the present problem. It has been shown that resistances sensitive to temperature can be made so narrow that even in a very pure spectrum they do not exceed in width the breadth of the spectral lines, and that the delicacy of this method far exceeds that of photographic processes.

After a long series of experiments, platinum was chosen as the material for the bolometer resistances. This metal, it is true, is never to be obtained in such a degree of purity[1] that its temperature coefficient equals that of silver, copper, tin, or iron, but it possesses, on the other hand, in its durability and convenience of working, such superior advantages over all other materials, that the loss in sensitiveness, conditioned by its smaller temperature coefficient, is more than compensated.

Professor Edward L. Nichols, of Cornell University, very kindly supplied me with a quantity of platinum wire drawn in silver,

[1] The smallest traces of iridium seriously affect the result.

from the workshops of W. and L. E. Gurley, of Troy, N.Y. While still in its covering of silver, this fine platinum wire, only .005 mm. in diameter, was hammered flat on a polished anvil, after which the silver was dissolved with nitric acid from the platinum ribbon within. The platinum strip so formed had a breadth of .05 mm. From this and its slightly increased length due to the hammering its thickness was calculated to be .00036 mm.

Two platinum threads prepared in this way were fastened side by side with shellac upon a little frame of mica. The frame itself was then fastened in such a way in a circular wooden case that one of the strips, which had previously been blackened by smoking in a turpentine flame, came directly opposite a rectangular opening in the cover, and was thus exposed to all radiations, while the other was completely protected by the walls of the case. Corresponding to the size of the opening in the diaphragm of the casing (3 × 7 mm.), the platinum threads, when in position, had a length of 7 mm.

Figure 1 represents the arrangement of the interior of the little wooden box. Copper wires, soldered to the ends of the platinum threads, connected these strips with the three brass sleeves *A*, *B*, *C*. These sleeves were then slipped over three corresponding rods in the bolometer sheathing or holder, and thus served to support the little case in its place, and at the same time to afford electrical contact with the interior. The points *A* and *C* led to the battery, while *B* formed one point of contact with the galvanometer.

Fig. 1.

The two resistances were made as nearly alike as possible, and each measured about 75 ohms. The other two resistances in the Wheatstone's bridge were made of fine German silver wire, and wound non-inductively upon corks.

It was soon found that the task of balancing the Wheatstone's bridge would be made much lighter if small changes could be introduced at pleasure in the relation of the resistances forming the four arms. To attain this result, the two branches of the bridge last named, made of German silver, were connected by a German silver wire 50 cm. long and .5 mm. in diameter, upon which rested a sliding contact.

A short trial, however, soon proved that a simple displacement of this point of contact was not well adapted to a rapid and accurate balancing of the bridge. With the exceedingly high degree of sensitiveness of the galvanometer employed, a change in the position of the sliding contact of a fraction of a millimeter caused a throw of the galvanometer needle of several meters, a circumstance which rendered the manipulation of the apparatus exceedingly difficult. There was accordingly placed upon the bridge wire MM' (Fig. 2) a second movable contact, and both of these sliding contacts, p and q, were joined with the ends, P and Q, of a straight platinum wire, upon which slipped a small glass T-shaped tube filled with mercury. The latter formed the other terminal of the galvanometer circuit. AB and BC were the two platinum resistances in the circular case, and AM and CM' the other two resistances, joined by the bridge wire MM'. The battery connection was made at points A and C.

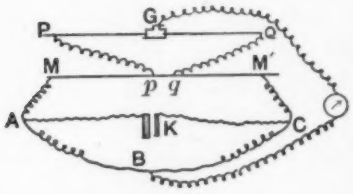


Fig. 2.

With this apparatus, the balancing of the bridge was effected in the following manner: The sliding contacts, p and q, were first placed in such a position that the point which had the same potential as the point B should fall between them. The distance between these movable contacts was then made as small as possible. Since the resistance of the connecting wires, pP and qQ, was negligible in comparison with that of the bridge wire, there existed the same fall of potential between the points P and Q as between the points p and q. It was possible, therefore, with this arrangement to vary at will the sensitiveness of the apparatus within wide limits. The approximate adjustment was made by displacing p and q, and the final balancing accomplished by slipping the tube G along the wire PQ.

To determine the sensitiveness of the instrument to thermal changes, a method was employed which has been described by Rubens and Ritter.[1]

[1] *Rubens* and *Ritter*, Wied. Ann., 40, p. 62, 1890.

In parallel arc with one of the bolometer resistances of 75 ohms was placed a shunt of 1000 ohms, and around two ohms of this resistance was laid a second shunt of 100 ohms (see Fig. 3). In the latter circuit was placed a key. Opening or closing this key changed the resistance of the first shunt circuit by nearly $\frac{2}{1000} \cdot \frac{2}{100}$ of its former amount, whereby the entire resistance of the arm of the bolometer suffered a change amounting nearly to $\frac{75}{1000} \cdot \frac{2}{1000} \cdot \frac{2}{100}$. The temperature coefficient of the platinum employed being .0030, the sensitiveness to temperature, as calculated from the above figures and the throw a, of the galvanometer, was found to be

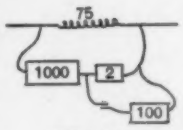


Fig. 3.

$$k = \frac{75}{1000} \cdot \frac{2}{1000} \cdot \frac{2}{100} \cdot \frac{1}{.0030\,a}.$$

With the degree of astaticism used throughout the experiments, a was about 130 mm. The constant k was therefore found to be

$$k = \tfrac{1}{130000}{}^{\circ}\,\mathrm{C.}$$

With this sensitiveness, a standard candle at a distance of 1 m. produced a deflection of 150 mm.

2. *The Galvanometer.*

It is well known that it is not difficult to make a bolometer sufficiently delicate to indicate minute differences of temperature, provided the surface exposed to the radiation may be made sufficiently large. If, however, it is desired, as in the present investigation, to discern fine lines in the spectrum by the heat which they emit, it is necessary to use for the purpose a linear bolometer in which the surface upon which the radiations fall comprises only a small fraction of a square millimeter. The illuminated filament of platinum in the bolometer already described, being 7 mm. in length and .05 mm. in breadth, exposed a surface of about $\frac{1}{3}$ sq. mm.

Since the sensitiveness of a bolometer is nearly proportional to the square root of its available surface, it was necessary to use a galvanometer of the highest degree of delicacy in order that the

deflections might be sufficiently large to render the observations trustworthy. After many fruitless attempts to make use of the instruments already at hand, it was decided to construct one which would have the required degree of sensitiveness. Since a delicate galvanometer is often required in other lines of work, it may be not altogether without profit to give the details of its construction.

In order to obtain the highest efficiency from the galvanometer, it is necessary to use an astatic system which combines the smallest possible moment of inertia with the greatest strength and uniformity of magnetization. These two purposes were accomplished in the following arrangement.

After the model of Lord Kelvin's instruments, the magnets consisted of twelve little steel bars, 3 and 4 mm. long, fastened with shellac to the ends of an exceedingly fine, thin-walled capillary tube, three magnets being on the front side of the tube, and three on the back, at each end. A very small watch spring furnished the steel from which the magnets were made. This was first heated in potassium ferrocyanide and then hardened in mercury. Half-way between the two systems of magnets, and resting in its little stirrup of tissue paper, was placed a small mirror 5 mm. in diameter and .14 mm. thick.

Fig. 4.

To make a mirror sufficiently accurate for the purpose, about fifty microscope cover-glasses were cut in pieces 5 mm. square, cleaned, and laid upon a piece of plane glass. These little plates were then viewed by the monochromatic light of the sodium flame, and the interference figures observed which were produced in the enclosed layer of air. It was found possible to tell by the form and position of these interference bands which pieces were adapted to the manufacture of mirrors.

Such interference figures are in general elliptically or hyperbolically curved, thus showing that the surface under examination has two noticeably different radii of curvature. It is clear that such surfaces can give no perfect images, inasmuch as an astigmatic eye would be necessary to see objects in their natural form.

If, on the other hand, the interference bands form systems

of parallel straight lines or concentric rings, we have to deal with plane surfaces or surfaces of revolution, which are, in most cases, capable of giving good definition.

A number of these pieces were thus selected from the rest, silvered, and examined with a telescope and scale. Finally, that one which seemed the most nearly perfect was fastened, as already described, upon the needle of the galvanometer.

The weight of the entire needle, including the twelve magnets, the mirror, holder, and connecting rod, was about 80 milligrams.

As a suspension for the needle, an exceedingly fine quartz fiber, 40 cm. long, was employed, instead of one of the fibers 10 c.m. long which Professor C. V. Boys very kindly sent for the purpose. It seemed at that time that the torsion of the latter would be too great when compared with the very feeble directive force employed. This was, however, an erroneous belief, as was abundantly proven by later experiments.

Next to the construction of the needle, the greatest care was given to the design of the coils. Particularly did it seem of the greatest moment to make the best possible use of the space available for winding the wire. With this in view, several points were to be considered. The end to be attained demanded that the coils be given such a form that a current of a certain strength, and meeting within the coil the smallest possible resistance, should exert upon the needle the maximum magnetic effect. A simple consideration shows[1] that the space to be filled with wire must be given such a form that its surface will be bounded by the magnetic lines of force due to the current. The effectiveness will be still further increased if the wire is brought as close as possible to the needle.

In order to conform to these several conditions, a device was employed by which it was possible completely to dispense with the spool or thimble upon which the wire is usually wound, and at the same time to give the coils approximately the desired shape.

These ends were accomplished by providing a brass block,

[1] *Maxwell*, Electricity and Magnetism, second ed., Art. 717; *Ayrton*, *Mather* and *Sumpner*, Phil. Mag. (5), 30, p. 63, 1890.

accfdb (shown in axial section in Fig. 5), the cylindrical part being 3 cm. in diameter, and the conical end at *ab* being 6 mm. in diameter. Upon this block was fastened, by the set screw *G*, the brass collar *H*. Against this ring as a shoulder rested the brass plate *CD*, having in its center a circular opening which allowed the plate to be supported upon the cylindrical part of the block. At the conical end, *ab*, a screw *S* held in place the plate *AB*. These two plates were then situated 15 mm. apart.

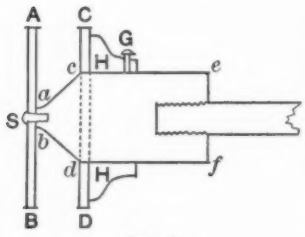


Fig. 5.

The space enclosed between this form and the two plates was then wound with wire. The part as far as *cd* contained 1100 turns of wire .25 mm. in diameter, and the remaining space was filled with 700 turns of wire .5 mm. in diameter.

The wire, while still in position between the plates upon the brass form, was then placed in a bath of melted paraffine, in which it remained until thoroughly heated. The vessel with all its contents was next placed under the receiver of an air-pump and the air withdrawn. As the vacuum became higher, the air which had been confined in the silk covering of the wire escaped, while at the same time the moisture was vaporized which had found its way, during the process of winding, to the insulating material around the wire. As soon as the atmosphere was again admitted to the receiver, the increased pressure caused the paraffine to fill the smallest interstices between the wire, so that, when cold, the whole formed a compact mass easy to handle.

The superfluous paraffine was then cut away, the screws *S* and *G* removed, and the plate *AB* quickly warmed in the flame of the Bunsen burner. Before the heat had been communicated to the remainder of the material, the plate slipped away from the wire, which still remained imbedded in the paraffine. The blast-lamp was then directed against the brass block, which in like manner became hot, and was removed without injury to the wire.

The coil in this form, held together as it was only by the

paraffine, and supported solely by the rear plate *CD*, was now ready for use in the construction of the galvanometer.

Two coils made in this way were mounted in a rectangular case directly below two similar coils (see Fig. 6), and so placed that the surfaces which, in the process of construction, had been formed by the plate *AB*, were now situated opposite each other and 1 mm. apart. The front and the rear sides of the case, upon which the coils were mounted, were made of wood, the other two sides being formed simply of narrow panes of glass. It was easy, there-

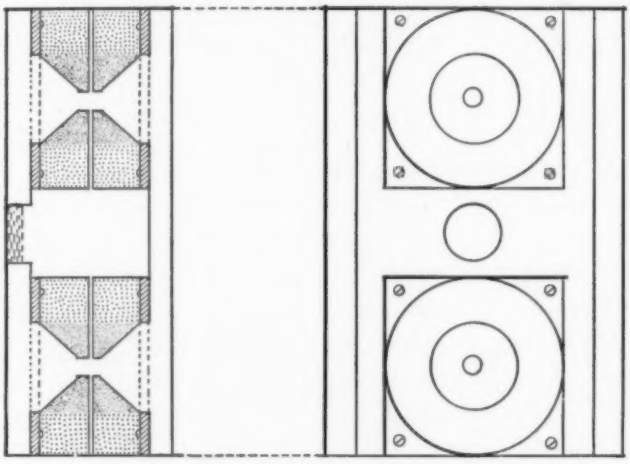


Fig. 6.

fore, to examine from the outside the condition of the interior, so that any lack of freedom in the suspended system could readily be corrected. Binding-screws, placed upon the front wall of the galvanometer case, formed the means of electrical contact with the coils within. That part of the wall which supported the two front coils was movable, and could be taken out and replaced at pleasure. The interior was thus at all times accessible.

The description of the galvanometer will now be complete when mention is made of the fact that the instrument was placed upon a base, provided in the usual way with three levelling-screws, and that upon the upper part of the case was secured a glass tube 40 cm. long, at the upper end of which was fastened the quartz thread for the suspension.

The characteristic constants of this galvanometer were as follows: When the coils were connected in series, the current traversed 7200 turns of wire, of which the total resistance was 140 ohms. When the controlling magnet near the instrument was in such a position that a double vibration of the needle was performed in 20 seconds, 1 mm. deflection on the scale 3 m. away indicated a current of 1.5×10^{-11} ampères. In conducting the final experiments, however, it was found that a sensitiveness less than this was amply sufficient, and the time of vibration was accordingly reduced to 7 seconds. Even then its delicacy was notably higher than that of the commercial instruments.

3. *The Calibration of the Prism.*

A prism of medium silicate-flint glass of high dispersive power, mounted upon a spectrometer from the workshops of Schmidt and Haensch, formed the apparatus with which the spectra were studied. The prism was preferred for this purpose to a diffraction grating, because the superposition of the spectra produced by the latter rendered difficult the detection and identification of lines in the infra-red, and further because the energy in the prismatic spectrum is far more intense than is developed in any one of the numerous diffraction spectra.

After the method recently described[1] by Rubens, a number of calibrations of the prism were made, in all of which Dr. Rubens himself kindly assisted. The several experiments yielded results agreeing well among themselves, from which the dependence of the wave-length of the infra-red rays upon the angle of deviation could be determined within about five parts in a thousand.

The results of the first calibration are shown graphically in Fig. 1, Plate I. This figure represents the energy spectrum of the source of light, crossed by the heavy interference bands. The double thickness of the layer of air producing the interference, multiplied by the cosine of the angle of incidence i of the rays, gave $k=6.677\mu$, as was determined from observations in the visible spectrum. The first infra-red interference band was

[1] *Rubens*, Wied. Ann., 45, p. 238, 1892.

of the 9th order, from which, accordingly, for this band, $n=9$. With the aid of these constants and the observed position of the maxima and minima, corrected before using by the method of envelopes given by Rubens, were calculated the values of the wave-lengths corresponding to the following angles of deviation ϕ.

TABLE I.

NAME.	ϕ	λ	NAME.	ϕ	λ
H_γ	50° 51½′	0.434 μ	b_3	46° 49′	1.026 μ
F	49° 46½′	0.486 "	a_4	39½′	1.111 "
D	48° 31½′	0.589 "	b_4	30′	1.214 "
C	2½′	0.656 "	a_5	20½′	1.336 "
a_1	47° 37′	0.753 "	b_5	9′	1.482 "
b_1	26½′	0.786 "	a_6	45° 55½′	1.669 "
a_2	16′	0.834 "	b_6	37½′	1.906 "
b_2	7′	0.889 "	a_7	14½′	2.225 "
a_3	46° 58′	0.952 "	b_7	44° 45½′	2.668 "

Two other series of observations were conducted in precisely the same way, except that the thickness of the layer of air which caused the interference was changed, so there were obtained in all three entirely independent calibrations of the prism. The wave-lengths corresponding to a series of equidistant deviations of the bolometer arm were determined by interpolation from each of these curves. In Table II. are contained the results which were thus obtained. The first column contains the angle ϕ, the following three the wave-lengths taken from the respective curves of dispersion, and the last the mean of these results. From the values contained in the last column, and the corresponding angles of dispersion, the curve of dispersion (Fig. 2, Plate I.) was constructed. This curve was used in all the determinations of wave-length which follow.

Since the dispersion in the visible spectrum was very great, this region was calibrated separately by reference to a large number of Fraunhofer lines, and a curve of dispersion for this portion of the spectrum was plotted on a larger scale than that used for the infra-red.

TABLE II.

ϕ	I.	II.	III.	MEAN λ.
50° 51′	0.434 μ	0.434 μ	0.434 μ	0.434 μ
49° 46′	0.486 "	0.486 "	0.486 "	0.486 "
48° 31½′	0.589 "	0.589 "	0.589 "	0.589 "
2′	0.656 "	0.656 "	0.656 "	0.656 "
47° 50′	0.695 "	0.690 "	0.686 "	0.690 "
40′	0.729 "	0.722 "	0.723 "	0.725 "
30′	0.767 "	0.761 "	0.766 "	0.765 "
20′	0.813 "	0.809 "	0.814 "	0.812 "
10′	0.872 "	0.864 "	0.872 "	0.869 "
0′	0.940 "	0.937 "	0.945 "	0.941 "
46° 50′	1.018 "	1.014 "	1.022 "	1.018 "
40′	1.109 "	1.104 "	1.112 "	1.108 "
30′	1.215 "	1.218 "	1.222 "	1.218 "
20′	1.339 "	1.347 "	1.348 "	1.335 "
10′	1.473 "	1.481 "	1.482 "	1.479 "
0′	1.611 "	1.610 "	1.617 "	1.613 "
45° 50′	1.746 "	1.743 "	1.751 "	1.747 "
40′	1.881 "	1.872 "	1.887 "	1.880 "
30′	2.017 "	2.004 "	2.022 "	2.014 "

4. *On the Manipulation of the Bolometer.*

Before mention is made of the details and results of the experiments themselves, a few words of suggestion should not be omitted which may be found useful in the manipulation of a bolometer.

As already stated in an earlier part of the paper, two arms of the bolometer were made of German silver, and the other two of platinum. Of the two latter branches, the one which was exposed to the radiations had been coated with lamp-black by smoking in a flame, and thus through this exposure to heat its temperature coefficient had to some extent been changed. As a result of this slight difference in the condition of the two arms, a perfect equilibrium could be maintained within the Wheatstone's bridge at a single temperature only. If, accordingly, the temperature of the room rose or fell, a drift of the needle in the one direction or the other became at once perceptible.

The disturbing effects of this drift were almost completely avoided if, some time before the observations were to begin, the precaution was taken to close the circuit through the bolometer, to light the burners illuminating the galvanometer scale,[1] to set in operation the electric lamp, the purpose of which will be described later, and at the same time to open the door of the room, in order that as regards all thermal changes an equilibrium might be established.

Even with the utmost care it was impossible to entirely eliminate the drift of the needle, but it was easily possible to make this disturbance so small that for an interval of many minutes the cross-hairs remained upon the same division of the scale. So sensitive was the apparatus to any changes in the temperature of the room, that it was invariably found possible to reverse the direction of the slow, ever-present drift simply by increasing or decreasing, often by not more than 1 cm., the amount by which the door stood open. This simple but effectual regulation frequently made it an easy matter to conduct the entire series of observations, lasting three hours, without altering the position of the sliding contact G (Fig. 2) in the Wheatstone's bridge.

The effect of the feeble drift which even this expedient failed to remove was still further reduced by observing the zero point of the needle after the throw as well as before, and taking the mean as the true zero. By this method of observation a uniform drift would be wholly without influence upon the result.

In the entire series of measurements it was not the final deflection, but the first throw of the needle, which was observed. There was secured by this means not only a saving of no inconsiderable amount of time, but also a greater freedom from the mechanical, thermal, and magnetic changes to which, notwithstanding all precautions to the contrary, a crowded laboratory, situated in the midst of a large city, is constantly exposed.

In a recent paper,[2] Professor Merritt has shown that the first

[1] The scale was removed from the telescope and placed about 3 m. from the bolometer, from which it was separated by a cardboard screen.

[2] *Merritt*, Am. Journal of Science (3), 41, p. 422, 1891.

throw of a galvanometer, when used with a thermopile or a bolometer, bears a constant ratio to the final deflection, and is therefore proportional to the heat radiated.

At each position throughout the spectrum the energy was measured by two such observations and the mean taken.

III. THE SPECTRUM OF THE ELECTRIC ARC.

Many vain attempts were made to detect traces of lines in the infra-red spectra of the alkalies when these metals were burned in a Bunsen burner or in the oxyhydrogen flame. Not until the electric arc was used for this purpose did efforts in this direction meet with success.

Inasmuch as it was suspected that a number of the observed infra-red spectral lines owed their origin to the arc itself and not to the metals therein consumed, it seemed all important to postpone the study of metallic spectra until a careful investigation of the distribution of energy in the infra-red spectrum of the electric arc had been made.

As a means of producing the arc an old Duboscq lamp was used, which, when supplied with current from the constant potential mains of the Berlin Central Station (110 volts), burned very quietly. By introducing resistance the current through the lamp was maintained constant at from 7.5 to 7.8 ampères.

After many experiments with carbons of various sizes, it was found that more perfect uniformity in the length of the arc ($6\frac{1}{2}$ mm.), together with greater quietness and regularity of burning, were secured by using carbons 8 mm. in diameter. The arc exhibited also less of a tendency to wander when a cored carbon was used as the anode.

By means of a lens, a real image of the arc, enlarged about twofold, was projected upon the slit of the spectrometer. Immediately before the slit was placed a large screen in such a way that the images of the incandescent carbons, as well as the lateral portions of the arc itself, were intercepted, and only a small strip, 7 mm. long and exceedingly narrow, belonging to

the central region of the projected arc, was allowed to enter the collimating tube of the instrument.

In the entire series of measurements which follow, the opening of the slit did not exceed .1 mm. in width, and subtended an angle in the spectrum of 1.68 minutes. The width of the slit had been adjusted until its image in the telescope was just superposed upon the thread of the bolometer with its lamp-black coating.

Although the strength of the current in the Wheatstone's bridge was maintained as nearly constant as possible at $\frac{1}{40}$ ampère, yet before each series of observations the sensitiveness of the apparatus to temperature was determined according to the method already described, so that the results of the different determinations could be compared.

The energy was always assumed to be proportional to the deflection of the galvanometer needle. This assumption, however, is justified by the papers of Ångström,[1] Rubens,[2] and Merritt.[3]

In the manner already described, a large number of observations were conducted in order to determine the character of the spectrum of the arc light. The almost perfect agreement of the results obtained makes it a matter of indifference which particular series is selected from the material at hand to be given here in all its details.

The data are given with greater completeness in the following table than in the subsequent measurements, from the fact that the same order of procedure was followed throughout the entire investigation, and in the description of the distribution of energy in metallic spectra, to be given later, reference will be made to this series for many minor points.

The first column contains the angular deviations of the arm of the bolometer as read on the circle of the spectrometer, and the second contains the corresponding wave-lengths, determined with the aid of the curve of dispersion, Fig. 2, Plate I. In the third column are found the means of the galvanometer deflections,

[1] *Ångström*, Wied. Ann., **26**, p. 262, 1885.

[2] *Rubens*, Wied. Ann., **37**, p. 256, 1889.

[3] *Merritt*, Am. Journal (3), **37**, p. 167, 1889. *Merritt*, Am. Journal (3), **41**, p. 422, 1891.

which were taken in the manner described above, and which were used without further correction as the relative values of the energy measured. (See Table III. on pages 45 and 46.)

Sensitiveness of the Apparatus to Temperature. — Deflections: 128, 135, 128, 132, 132, 134. Mean: 132.

$$a = \frac{136 + 132}{2} = 134.$$

Wherefore $k = \frac{1}{134000}$° C.

The data contained in this table are graphically represented in the curve Fig. 3, Plate I. Angular deviations of the arm of the bolometer are chosen as abscissæ, and corresponding deflections in millimeters as ordinates.

In order that the positions of the maxima and minima of this curve may be more easily recognised, a number of Fraunhofer lines are drawn for comparison. The limits of the visible spectrum, as ordinarily accepted, are indicated by dotted lines. The assignment of these limits is naturally attended by much uncertainty, since the investigations of Helmholtz,[1] Esselbach,[2] Becquerel,[3] and Langley[4] have shown that under peculiarly favorable conditions the eye can perceive radiations, the wave-lengths of which lie far beyond these boundaries. The present paper also contains facts tending to confirm these statements.

In Fig. 4, Plate I., the same curve is represented, with the single exception that wave-lengths are chosen as abscissæ instead of divisions on the circle.

A glance at these curves shows at once the remarkable fact that *in the spectrum of the electric arc, the maximum of the energy lies farther in the direction of the short wave-lengths than the Fraunhofer lines H and K.*

The principal maximum of the energy curve extends from

[1] *Helmholtz*, Pogg. Ann., **94**, p. 12, 1855.

[2] *Esselbach*, Monatsber. d. k. Acad. d. Wiss. zu Berlin, p. 757, 1855; *Esselbach*, Pogg. Ann., **98**, p. 514, 1856.

[3] *H. Becquerel*, Compt. Rend., **97**, p. 73, 1883.

[4] *Langley*, Phil. Mag. (5), **21**, p. 396, 1886.

Table III.

ELECTRIC ARC.

Sensitiveness of the Apparatus to Temperature. Deflection in Millimeters: 137, 138, 138, 130, 136. Mean: 136.

Observations of Energy.

ϕ	λ	I	ϕ	λ	I
54° 7½′		0	49° 56½′	0.4770 μ	3
			49′	0.4838 "	3
53° 57′		0	41′	0.4920 "	2
50½′	0.365(?)μ	11	21′	0.5140 "	9
44½′		0	19½′	0.5155 "	8
4½′	0.3770 "	13	15′	0.5213 "	4
52° 39½′	0.3820 "	80	48° 51½′	0.5540 "	6
29½′	0.3850 "	185	34½′	0.5835 "	9
27½′	0.3858 "	284	31½′	0.5892 "	14
26′	0.3861 "	306	28½′	0.5945 "	11
24½′	0.3868 "	279	26′	0.600 "	11
23′	0.3871 "	310	21′	0.611 "	12
22′	0.3874 "	282	17½′	0.619 "	13
20½′	0.3880 "	328	14′	0.627 "	15
19½′	0.3883 "	357	11′	0.635 "	18
18′	0.3889 "	83	8½′	0.642 "	19
16½′	0.3894 "	29	5½′	0.649 "	21
10½′	0.3918 "	24	2½′	0.657 "	20
51° 48½′	0.4015 "	10	47° 59½′	0.666 "	20
30½′	0.4110 "	29	57′	0.673 "	21
27′	0.4128 "	48	55′	0.679 "	17
24½′	0.4140 "	76	53′	0.685 "	22
23′	0.4150 "	72	50½′	0.693 "	26
22′	0.4157 "	64	48′	0.701 "	26
21′	0.4163 "	68	45′	0.711 "	36
19½′	0.4170 "	68	43′	0.718 "	35
18½′	0.4177 "	54	41′	0.726 "	40
17′	0.4183 "	62	39½′	0.731 "	43
15½′	0.4194 "	35	38′	0.737 "	39
14′	0.4203 "	42	36½′	0.743 "	39
11½′	0.4210 "	8	35′	0.749 "	41
½′	0.4250 "	4	33½′	0.755 "	32
			32′	0.761 "	37
50° 31′	0.4488 "	7	30½′	0.768 "	35
28′	0.4508 "	12	29′	0.775 "	30
25′	0.4532 "	6	27½′	0.781 "	30

TABLE III. — *Continued.*

ϕ	λ	I	ϕ	λ	I
47° 26′	0.788 μ	50	46° 25′	1.276 μ	12
24½′	0.794 "	57	23½′	1.295 "	8
23′	0.801 "	62	22′	1.312 "	5
21½′	0.808 "	74	20½′	1.330 "	5
20′	0.815 "	68	19′	1.352 "	8
18½′	0.823 "	66	17½′	1.372 "	13
17′	0.830 "	65	16′	1.392 "	21
15½′	0.838 "	54	14½′	1.413 "	24
14′	0.846 "	51	13′	1.435 "	24
12½′	0.854 "	44	11½′	1.453 "	24
10½′	0.865 "	33	9½′	1.482 "	21
9′	0.875 "	30	8′	1.502 "	14
7½′	0.886 "	26	6½′	1.523 "	11
6′	0.896 "	23	5′	1.543 "	10
4½′	0.906 "	60	3½′	1.563 "	8
3′	0.917 "	98	2′	1.583 "	9
1½′	0.928 "	95	½′	1.603 "	6
0′	0.940 "	97			
			45° 59′	1.626 "	4
46° 58½′	0.951 "	86	57½′	1.645 "	3
57′	0.962 "	67	56′	1.665 "	2
55½′	0.974 "	57	54½′	1.685 "	2
54′	0.987 "	48	53′	1.705 "	2
52½′	0.999 "	36	51½′	1.725 "	0
51′	1.012 "	29	50′	1.747 "	0
49½′	1.024 "	24	48½′	1.769 "	1
48′	1.038 "	18	47′	1.789 "	0
46½′	1.050 "	16	45½′	1.810 "	0
44½′	1.068 "	16	44′	1.830 "	0
43′	1.082 "	75	42½′	1.850 "	0
41½′	1.096 "	103	41′	1.870 "	0
40′	1.109 "	80	39′	1.897 "	0
38½′	1.128 "	59	37½′	1.918 "	0
37′	1.144 "	47	36′	1.938 "	0
35½′	1.158 "	35	34½′	1.958 "	0
34′	1.176 "	29	33′	1.978 "	0
32½′	1.192 "	26	31½′	2.000 "	0
31′	1.208 "	23	30′	2.020 "	0
29½′	1.225 "	17	28½′	2.042 "	0
28′	1.243 "	18	27′	2.062 "	0
26½′	1.258 "	13			

$\lambda = .385\,\mu$ to $\lambda = .388\,\mu$; a secondary maximum is comprised within the region between $\lambda = .411\,\mu$ and $\lambda = .420\,\mu$.

In examining these exceedingly intense bands with the eye, it was found that the first consisted of five narrower bands, and the second of six, all of which were sharply defined on the side looking toward the red end of the spectrum, but which grew rapidly indistinct and hazy in the opposite direction.

A peculiarity of the arc spectrum was observed in this connection which, as was found later, had also been noticed by Kayser and Runge,[1] viz.: that when the slit of the spectroscope is made very narrow, the entire spectrum of the electric arc consists of many thousand fine lines which lie exceedingly close together at those places where the bright bands are visible, and are arranged at slightly greater intervals in the other portions of the spectrum.

With the bolometer, in the form in which it was used, it was not possible to detect these individual lines, which are often not more than a second of arc apart. Only a summation could be obtained which represents an integral of all the smaller portions of energy which fell upon the filament of the bolometer, 1.6 minutes of arc in width. The energy in the arc spectrum was accordingly measured with a breadth of slit approximately equal to the width of the sensitive bolometer thread. What was then observed was the characteristic banded spectrum, formed by a partial overlapping of this large number of fine lines.

Shortly before the close of the entire investigation, there was detected, first optically and then with the aid of the bolometer, a feeble band which lies still farther in the direction of the extreme violet than the group from $\lambda = .385\,\mu$ to $\lambda = .388\,\mu$. By an extensive extrapolation of the curve of dispersion an approximate wave-length, $\lambda = .365\,\mu$, was ascribed to this band, but this value, it must be remembered, lays no claim to being correct. Without doubt this band is identical with the one found by Kayser and Runge[2] to lie from $\lambda = .3584\,\mu$ to $\lambda = .3590\,\mu$.

In their spectrophotometric comparison of the various sources

[1] *Kayser* und *Runge*, Wied. Ann., **38**, p. 81, 1889.
[2] *Kayser* und *Runge*, Wied. Ann., **38**, p. 90, 1889.

of illumination, Nichols and Franklin[1] have described a band in the spectrum of the electric light, which can be easily identified with the series of bands from $\lambda=.4110\mu$ to $\lambda=.4210\mu$; for, although representing far less energy than the group between $\lambda=.385\mu$ and $\lambda=.388\mu$, the former are optically much brighter than the latter. It is also to be noticed that the limits for the wave-length of this band, as given by Nichols and Franklin, lie nearer the group between $\lambda=.4110\mu$ and $\lambda=.4210\mu$.

In a recent paper, P. Drude and W. Nernst[2] mention two bands in the spectrum of the arc light which excite a peculiarly active fluorescent effect upon a solution of fluorescene. They determined the wave-lengths of these bands with the aid of a diffraction grating, and obtained for the more refrangible and at the same time optically weaker band the value $\lambda=.386\mu$. This determination coincides so closely with the values found in the present paper for bands in this region as to leave no doubt as to the identity of those bands with the ones here investigated.

The energy throughout the remaining portions of the visible spectrum of the electric arc, as compared with the unusually large amount to be found in the violet, is quite unimportant. A small quantity of a sodium salt, always present as an impurity in the carbons, may be detected by the appearance of several of the characteristic sodium lines. The stronger of these, especially the D-lines, may be measured with the bolometer. In the visible spectrum are found, moreover, several indistinct bands which, on account of their feeble energy, are hardly to be distinguished from the bright background, and therefore are scarcely to be recognized in the figure. Not until the region of the infra-red is reached are well-developed maxima and minima again found, and here the bolometer detects five intensely warm bands, similar in many respects to those in the extreme violet, but showing, on the other hand, the noticeable difference that their sharply defined edges are turned toward the visible portion of the spectrum, while an indistinct border bounds each of these bands

[1] *Nichols* and *Franklin*, Am. Journal (3), **38**, p. 106, 1889.
[2] *Drude* und *Nernst*, Wied. Ann., **45**, p. 460, 1892.

on the side of the longer wave-lengths. In all probability these bands consist of a warm background upon which is superposed a series of lines more or less sharply defined. The dispersion of the prism employed was, however, too feeble to permit of a complete separation of these lines by the bolometer. Their presence was nevertheless detected by the small irregularities which the curve shows in the neighborhood of each of the five maxima.

The wave-lengths of these five infra-red bands are as follows:—

TABLE IV.

A.	from	$\lambda = 0.700\,\mu$	to	$\lambda = 0.770\,\mu$
B.	"	$\lambda = 0.785$ "	"	$\lambda = 0.860$ "
C.	"	$\lambda = 0.900$ "	"	$\lambda = 1.000$ "
D.	"	$\lambda = 1.075$ "	"	$\lambda = 1.160$ "
E.	"	$\lambda = 1.370$ "	"	$\lambda = 1.500$ "

IV. ON THE GRAPHICAL REPRESENTATION OF THE DISTRIBUTION OF ENERGY THROUGHOUT THE SPECTRUM.

In general it is not possible, in transforming to the normal spectrum the distribution of energy as observed by means of a prism, to retain in their original lengths the ordinates which in the diffraction spectrum represent the energy throughout the region measured. At each point of the curve, the lengths of the new ordinates must be so chosen that the curves (see Fig. 7), bounded by the wave-lengths λ and λ' and comprised between the interval $(\lambda - \lambda')$ on the axis of abscissæ and the portion of the curve immediately above, must, both in dispersion and diffraction spectrum, be equal.

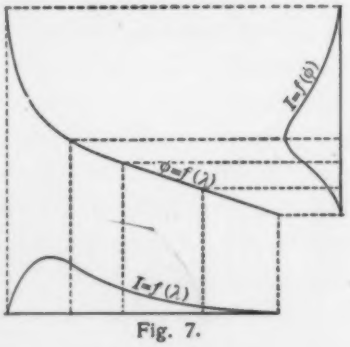


Fig. 7.

Professor Langley has suggested [1] a simple means of accomplishing this purpose, and illustrates his method by a drawing from which the idea of the accompanying figure has been taken.

[1] *Langley*, A Report of the Mt. Whitney Expedition, p. 231, 1884. Phil. Mag. (5), 17. p. 211, 1884.

While this method serves to transform to the normal a continuous prismatic spectrum, it fails in its application to a bright line or discontinuous spectrum. In the latter case the width of the lines, so long as these do not overlap, is entirely independent of the amount of dispersion produced by the prism, and the reduction to the normal spectrum can be made without changing the lengths of the ordinates.

With the aid of a micrometer eye-piece it was determined that the width of the sharp spectral lines, for the amount of dispersion used in these determinations, was the same throughout all regions of the visible spectrum. Nevertheless, it was obviously impossible to consider the distribution of energy, shown in Fig. 4, Plate I, as the distribution in the normal spectrum, inasmuch as the lines, of which the entire series of bands consists, to a greater or less extent overlap. Moreover, the spectrum, strictly speaking, is not a continuous one, and therefore the reduction, as given by Langley, cannot be here applied. In the delineation of the energy contained in the radiations from the electric arc as well as in that of the metals to be later described, the author has been compelled to use as ordinates the values obtained from the dispersion spectrum, and to plot wave-lengths as abscissæ instead of angular deviations.

Mention must be made at this point, however, of the fact that if the assumption be made that the spectrum of the electric arc is a continuous one, in the sense in which the spectrum of an incandescent solid is so considered, the violet bands at $\lambda = .388\,\mu$ and $\lambda = .415\,\mu$ would be many times more intense, as compared with the energy in the infra-red bands, than appears in the drawing, Fig. 4, Plate I.

[*To be concluded.*]

THE CRITICAL CURRENT DENSITY FOR COPPER DEPOSITION AND THE ABSOLUTE VELOCITY OF MIGRATION OF COPPER IONS.

By Samuel Sheldon and G. M. Downing.

IT is a well-known fact that, in the electro-deposition of copper from solutions of its salts, if with a given exposed surface of cathode a certain current strength be exceeded, the deposit assumes a chocolate brown color when the solution is concentrated, or a dead black color when the solution is dilute. It is often stated that this modified form of deposit is but copper in a finely divided state. Magnus,[1] however, long ago stated that it was hydride of copper. He assumed that, for some reason or other, after a certain *critical current density* was reached, either the water of solution or H_2SO_4, formed by the union of liberated SO_4 with the water, took part in the conduction near the cathode. He endeavored to measure this current density by varying the current which was passing through an electrolytic cell containing $CuSO_4$ and then noting the strength at the moment when the hydrogen bubbles began to appear at the cathode. His results were not very concordant. C. L. Weber[2] explained the appearance of the copper hydride as follows: as the current which is passing through a certain copper solution with definite sized electrodes increases, the rate of deposit increases, and as a result, the rate of abstraction of copper from the portion of the solution around the cathode. Inasmuch as the copper ions have a definite absolute velocity, this rate of deposit is limited. If the current strength demands a greater deposit, the SO_4 ions, at the moment

[1] Magnus, Pogg. Ann., 112, p. 23, 1857.
[2] Ztschr. f. phys. Chem., 4, pp. 182–188, 1889.

of electrolytic severance from the copper, join with the water, form H_2SO_4, and this takes part in the conduction, depositing hydrogen alongside of the copper. Weber endeavored to measure the absolute ionic velocity of the copper ions. He passed a current through a variable resistance, a galvanometer, and a solution contained in a glass tube. The electrodes were pieces of copper, ground to fit the ends of the tube. The potential difference between the electrodes was measured by means of a high resistance galvanometer. The current was increased by steps and closed for 15 seconds. The galvanometer deflections were read just before the one of high resistance indicated a sudden rise in the potential difference, owing to the polarization resulting from the newly deposited hydrogen. It is not necessary to give his method of calculating the ionic velocities, for it is analogous to the one to be explained later. His results were not wholly satisfactory, errors of 10 per cent being admitted.

The present investigation considers the causes which affect the critical current density for copper sulphate solutions, and includes a determination of the absolute velocities of migration of the copper ions in solutions of varying concentration.

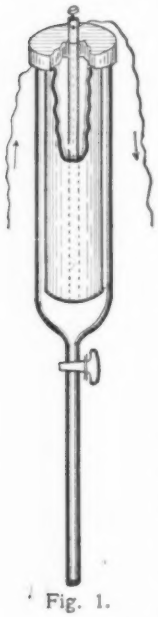


Fig. 1.

Apparatus. — The current was taken from the Edison street mains at 115 volts, and its strength was regulated by a bank of lamps. The current was measured by a Weston 0–5 ampère ammeter. The potential differences were measured by a Weston 0–150 volts voltmeter, whose series high resistance coil was short-circuited. A resistance of 314 ohms was inserted in the voltmeter circuit, causing the pointer to deflect the whole 150 scale divisions for 3 volts. The electrolyte was contained in a glass vessel of shape indicated in Fig. 1. The anode was a hollow copper cylinder of 4 cms. internal diameter. The cathode was a copper-plated brass rod of .648 cm. diameter. This was fastened into the center of a wooden cover, grooved to fit exactly over the top of the glass vessel.

Method. — The solution to be investigated was placed in the vessel and a current of somewhat less than the critical strength was sent through. This produced the normal copper deposit. The stopcock at the bottom of the vessel was then opened, and as the electrolyte ran out, the cathode surface diminished. Inasmuch as the resistance offered by the electrolyte was very small compared to that of the whole circuit, there was very little alteration in the current strength during the flow. Accordingly, after a certain amount of the electrolyte had flowed out, the surface of the cathode was reduced so as to produce the critical current density. The deposit immediately assumed the dark color, and upon removal of the cathode, the area could be readily measured by means of a micrometer gauge and a scale. The line separating the reguline deposit from the dark-colored one was very distinct, especially when produced from a moderately dilute solution. In all cases its position could be determined within .3 mm. The current was generally adjusted so that the height of the dark deposit was about 125 mm.

Influence of Rapidity of Flow. — Inasmuch as the appearance of the dark deposit is intimately associated with the alteration in density of the solution near the cathode, it is evident that the flow, upon turning the stopcock, must be so regulated as not to disturb the solution at this place. The form of apparatus was a happy one in this respect. This was proved by the following observations: A large quantity of $CuSO_4$ solution was prepared; portions of this were allowed to run, in succession, through the vessel at different rates. The current strengths (in ampères) divided by the areas (in square cms.) covered by the black deposit, *i.e.* the *critical current densities D*, are given in the following table, opposite to the times required to flow out of the vessel. The values of *D* are corrected for slight differences of temperature.

An inspection of this table shows that if the time which it takes the electrolyte to flow out of the vessel be greater than eight or nine minutes, the values of *D* will be practically the same as those which would be obtained were the electrolyte at rest.

TIME.		$D_{23°}$.
Min.	Sec.	
0	23	0.0886
0	34	0.0581
1	0	0.0455
1	30	0.0405
2	16	0.0375
3	3	0.0361
5	0	0.0342
8	49	0.0334
15	0	0.0330
20	0	0.0332

Influence of Temperature. — As might be expected, temperature has a great influence upon the value of the critical current density. Theory would indicate that the temperature coefficient should be the same as that for the specific conductivity of the solutions. Kohlrausch's[1] values referred to the conductivity at 18° are from .021 to .024; *i.e.* the specific conductivity at 18° increases for each degree rise of temperature from 2.1 to 2.4% according to concentration.

We have failed to obtain coefficients as small as these, ours ranging from .031 to .038. Our method of determination was to fill several glass-stoppered flasks with a solution taken from a stock bottle which had been previously shaken. These flasks were placed in water baths and brought to various temperatures, suffering no evaporation in the process. These solutions were successively placed in the density vessel, and as soon as the dark deposit appeared the temperature was taken. It seems hardly possible that the values given above are far from correct.

Influence of Concentration. — The critical current density increases with the concentration. We have investigated solutions containing 2, 1, .5, and .1 gram equivalents to the liter of solution. (A gram equivalent solution of $CuSO_4$ contains 31.55 grams of copper in a liter of the solution.) During preliminary observations the most concentrated solution became somewhat impover-

[1] F. Kohlrausch, Wied. Ann., **6**, 1 and 145, 1879.

ished of copper. Not all of the anions set free by the current were able to unite with the anode. Accordingly, after the final determinations, the specific gravity of the solution was obtained. This was also done with the other solutions. By means of tables[1] and the value given by Kohlrausch[2] for the specific gravity of a normal solution (1.0775 at 18°.2) we were enabled to determine the concentration.

Our results are given in the following table, where m represents the gram equivalents to the liter of solution, and D the current density in ampères per square centimeter. The values are reduced to 19°.

m	$D_{19°}$	D_{19}/m
1.89	0.1005	0.0532
1.	0.0528	0.0523
0.5	0.0257	0.0514
0.1	0.00542	0.0542

If the method of observation were susceptible of greater accuracy, it would undoubtedly be found that the values of D_{19}/m would increase with dilution in the same manner as the values of the specific molecular conductivity.[3]

For industrial or experimental purposes an approximation formula for determining the critical current density when the specific gravity of the solution is known may be of value. The excess of the specific gravity of solutions at 18°C above .9987 (the density of water at 18°) is very nearly proportional to the percentage of salt contained. Accordingly, if we represent the specific gravity of the solution by d, the critical current density in ampères per square centimeter of cathode surface

$$D_{18} = (d - .9987).65$$

within a few per cent. This is the limiting current allowable

[1] Landholdt und Börnstein, Tabellen, p. 147.
[2] F. Kohlrausch, Wiedemann's Ann., Bd. XXVI., p. 174, 1885.
[3] F. Kohlrausch, l.c., p. 196.

when the solution is at rest. By means of circulation at *the cathode surface* this value may be increased many times.

Velocity of the Copper Ions. — If the assumption be made that the cause of the abnormal hydride of copper deposit is the inability of the copper ions to migrate fast enough to keep up the supply for deposit, which is required by Faraday's law, the velocity of migration can be determined in the following manner. Suppose that a current of C ampères is the critical current for a deposit upon q square centimeters of cathode surface from a solution of m gram equivalents to the liter. Consider a layer of the solution around the cathode, of radial thickness dr_1. This layer contains $31.55\,mqdr_1$ milligrams of copper. The current requires that, in the short time dt, $.3281\,Cdt$ milligrams be deposited. If we suppose dr_1 to be of such a size that $31.55\,mqdr_1 = .3281\,Cdt$, and remember that C is of such a magnitude that the ions migrate with exactly the requisite velocity to maintain the reguline character of the deposit, it will be evident that all the copper ions in the elementary layer considered have migrated to the cathode in the time dt, and their velocity has been

$$V = \frac{dr_1}{dt} = \frac{.3281\,C}{31.55\,mq}. \qquad (1)$$

If the current were less than the critical amount C, the velocity would be smaller, as would also be the drop in potential along dr_1. In all cases when the current is less than C, the velocity is directly proportional to the drop in potential. The velocity in absolute measure must be expressed as so many centimeters per second under an impressed difference of potential of 1 volt per centimeter, measured along a line of current flow.[1] Let dE represent the volts of potential difference between the concentric surfaces enclosing the layer of thickness dr_1. Then the velocity of the copper ions in absolute measure,

$$V_0 = \frac{V}{\frac{dE}{dr_1}} = \frac{.3281\,Cdr_1}{31.55\,mqdE} \qquad (2)$$

[1] It must be remembered that the expression for the velocity of certain ions does not imply that all the individual ions migrate with that velocity, but that it is the expression for the mean value of their velocities.

Consider a single observation with the arrangement of apparatus which we have employed. Let

E = volts between electrodes,

r_1 = radius of cathode,

r_2 = radius of anode,

R = resistance of solution,

s = the specific resistance of the solution,

h = height of solution.

We have $E = CR$

and $$dE = CdR. \tag{3}$$

The resistance of an elementary hollow cylinder of the solution of radius r is

$$dR = \frac{sdr}{2\pi hr}. \tag{4}$$

Substituting this value in (3), we have

$$\frac{dE}{dr} = C\frac{s}{2\pi hr}. \tag{5}$$

If we consider $r = r_1$, we can substitute by merely adding the subscript to r in equation (2). But this expression is not in a convenient form for introduction into the expression for V_0, for it involves the value of the specific resistance of the solution. The formula can be altered so as to eliminate s, but it should be borne in mind that any lack of homogeneity in the solution during an observation, affecting s as it does, affects the validity of the application of the modified formula to the observations. By applying Ohm's law and integrating (4), we obtain

$$R = \frac{E}{C} = \int_{r_1}^{r_2} \frac{sdr}{2\pi hr} = \frac{s}{2\pi h} \log\left(\frac{r_2}{r_1}\right). \tag{6}$$

$$\therefore \frac{s}{2\pi h} = \frac{E}{C \log (r_2/r_1)}. \tag{7}$$

Substituting this value in (5) and again in (2), we have

$$V_0 = \frac{\cdot 3281\, Cr_1 \log (r_2/r_1)}{31.55\, mqE}. \tag{8}$$

In our experiments $r_1 = .324$ cm. and $r_2 = 2$ cm. Hence the formula reduces to

$$V_0 = .006136 \frac{C}{qmE}$$

The results which we have obtained are tabulated below, where m represents, as before, the gram equivalents to the liter of the $CuSO_4$ solutions.

m	C	q	E	V_0
1.89	2.59	25.16	1.55	0.000204
1.	1.35	25.56	1.26	0.000257
0.5	0.732	28.32	1.03	0.000308
0.1	0.128	22.78	0.775	0.000445

It is very probable that these values of V_0 are too large, owing to an increase in the specific resistance of the solution around the cathode, which follows the closing of the circuit. To eliminate this error as far as possible, in every observation the auxiliary resistances in circuit were so adjusted that the current, upon closing the circuit, was slightly less than the critical current for the exposed cathode surface. The surface was then decreased by allowing the fluid to flow out at the bottom of the vessel. The appearance of the abnormal deposit, indicated by a sudden rise in the voltage, followed the closing of the circuit in less than a minute.

The values given by Weber are less than the ones obtained by us. His method, too, involves the specific resistance and is open to the same criticisms as ours.

POLYTECHNIC INSTITUTE OF BROOKLYN,
March, 1893.

NOTE. GEOMETRICAL PROOF OF THE THREE-AMMETER METHOD OF MEASURING POWER.

By Frederick Bedell and Albert C. Crehore.

THE methods, now well-known, for measuring power by three voltmeters or three ammeters, first shown by Professor Ayrton and Dr. Sumpner,[1] are applicable to the measurement of power of any circuit irrespective of the nature of the impressed electromotive force, and the general proof of the methods is given in the paper referred to. In the case of an harmonic electromotive force the methods are capable of simple geometrical proof. The writers have shown this[2] for the voltmeter method, and in this note will give the corresponding proof for the three-ammeter method.

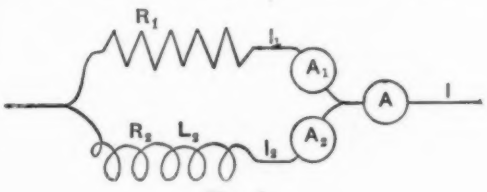


Fig. 1.

Let R_2L_2, Fig. 1, be an inductive circuit whose power is to be measured, and R_1 a non-inductive resistance in parallel with it. If the maximum values of the main and branch currents be denoted by I, I_1, and I_2, respectively, they may be represented as shown in Fig. 2. The current I_1 is in phase with the impressed electromotive force E; the main current, I, lags behind it by an angle θ; and the current I_2 lags behind it by an angle θ_2. The tangent of θ_2 is $\frac{L_2\omega}{R_2}$; and the tangent of θ is $\frac{L'\omega}{R'}$,

[1] "The Measurement of Power given by Any Electric Current to Any Circuit." Proc. Roy. Soc., Vol. XLIX., p. 424.

[2] "Alternating Currents," p. 230.

where R' and L' denote the equivalent resistance and self-induction of the parallel circuit,[1] and ω is 2π times the frequency.

The power expended in the inductive circuit is $W_2 = \frac{1}{2} EI \cos \theta_2$. From the geometry of the figure

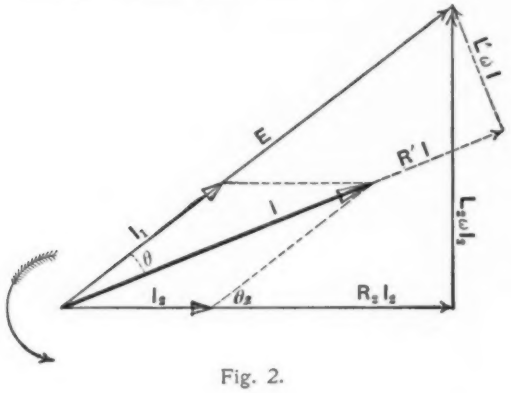


Fig. 2.

$$\cos \theta_2 = \frac{I^2 - I_1^2 - I_2^2}{2 I_1 I_2};$$

whence

$$W_2 = \frac{E}{4 I_1}(I^2 - I_1^2 - I_2^2),$$

where E, I, I_1, and I_2 represent maximum values. Writing virtual or square root of mean square values as obtained from ordinary measuring instruments, the expression for the power becomes:

$$W_2 = \frac{\overline{E}}{2 \overline{I}_1}(\overline{I}^2 - \overline{I}_1^2 - \overline{I}_2^2).$$

The power in the non-inductive branch is

$$W_1 = \overline{E}\,\overline{I}_1;$$

and the total power in the two branches is

$$W = \frac{\overline{E}}{2 \overline{I}_1}(\overline{I}^2 + \overline{I}_1^2 - \overline{I}_2^2).$$

[1] "Equivalent Resistance, Self-Induction and Capacity of Parallel Circuits with Harmonic Impressed Electromotive Force." Bedell and Crehore, Phil. Mag., September, 1892.

The method is thus geometrically established for harmonic currents, which may be represented by lines in a vector diagram. For an alternating current, not harmonic, the proof does not hold unless we assume the current to be equivalent to an harmonic current, and the question then arises as to what will be the equivalent harmonic current. The equivalent harmonic current must be such that its square root of the mean square value, and the expenditure of energy in the circuit, are the same as in the case of the given current which was not harmonic.

NOTES.

The International Congress of Electricians. — The committee appointed to arrange a programme for the World's Fair Congress of Electricians met in Washington on the 21st of April. There were in attendance T. C. Mendenhall, chairman, H. A. Rowland, H. S. Carhart, A. E. Kennelly, Elisha Gray, F. B. Crocker, and E. L. Nichols. It was decided that the Congress should be divided into three sections for the reading and discussion of papers: Section I., for the consideration of matters purely theoretical; Section II., of matters of mixed theory and practice; Section III., of matters purely practical. These sections may be further subdivided if they become unwieldly. Two sessions are to be held daily for the above purposes, in addition to which there will be three or four evening lectures and general sessions devoted to matters of wider interest to electricians. Papers on subjects covered by the three divisions already referred to are invited, the same to be submitted to the programme committee before August 1. The question of the acceptance of such papers for presentation will rest with sectional committees to be appointed later. The legislative work of the Congress, such as the establishment and legalization of electrical standards, will be performed by a special body consisting of five members from each country represented in the Congress. This body will be termed the Chamber of Delegates. It will report the result of its deliberations to Congress at a final sitting. The delegates appointed by the Secretary of State to represent the United States, are H. A. Rowland, T. C. Mendenhall, H. C. Carhart, Elihu Thomson, and E. L. Nichols.

Spring Meeting of the National Academy of Sciences. — The National Academy of Sciences held its regular spring meeting in the lecture hall of the National Museum, April 18, 19, and 20. Twenty-one papers were read either in full or by title, of which the following were of immediate interest to physicists: —

"On the Nature of Certain Solutions, and on a New Means of Investigating Them," by M. C. Lea.

"Telegraphic Gravity Determinations," by T. C. Mendenhall.

"Comparison of Latitude Determinations at Waikiki," by T. C. Mendenhall.

"A One Volt Standard Cell," by H. S. Carhart, introduced by T. C. Mendenhall.

"Fundamental Standards of Length and Mass," by T. C. Mendenhall.

"On the Potentiality of Internal Work in the Wind," by S. P. Langley.

"On the Bolograph in the Infra-Red Solar Spectrum," by S. P. Langley.

Professor Langley's papers were read by title.

Superintendent Mendenhall, in his paper upon gravity determinations, described an ingenious method of comparing the force of gravity at distant stations by the interchange of chronometer and pendulum signals. The method was given an excellent test in transferring the base for pendulum work from the Smithsonian Institute to the basement of the Coast Survey Building near the Capitol. The method is one which may be expected to give close comparisons between stations where telegraphic communication can be established.

Dr. Mendenhall's paper on the latitude of Waikiki was a summary of the careful investigations made at that station by Dr. Marchuse, representing the International Geodetic Association, and Mr. E. D. Preston, of the U. S. Coast Survey, in order to test the question of the periodic variation of latitude. The observed latitudes at two stations about forty feet apart were plotted for a period of more than a year. These show a marked and well-defined periodicity, the two curves running parallel to each other with a difference of ordinates corresponding closely to the difference in latitude of the two instruments. Both curves show a principal period of about 378 days, the oscillations being such as would be accounted for by a rotation of the earth's pole through a circle of forty feet in diameter.

In the discussion of this paper, Professor Chandler, who has paid much attention to the subject of fluctuations of latitude, pointed out that a reduction of observations obtained at Berlin and Waikiki gives curves which agree closely in amplitude, so that the question of the phenomenon's being local must be considered as settled. He pointed out the important effects of this movement upon various astronomical constants. It will be necessary to take cognizance of it in the zero point of right ascensions. The constant of nutation will be affected, and also that of aberration. There will result also an error in the accepted value of the precession, as well as in that of the obliquity of the ecliptic. In the opinion of Professor Chandler there is decided evidence of a third term in the equation of the curve which is perhaps periodic, with a period of from seven to fourteen years. It is of the highest importance in the further study of this problem that the old observations of the European and American observatories be worked over with a view to obtaining data covering long periods of time. What is most needed in the way of new work is the establishment of a competent observer in the southern hemisphere.

Professor Carhart described a standard cell of a type devised by Helmholtz, and since then employed by Ostwald. Carhart finds that the electromotive force of such cells can be controlled by changing the density of the solution, and that the cell can be adjusted so as to give precisely one volt. The form adopted is similar to Carhart's modification of the Clark cell. The elements are mercury, mercurous chloride, zinc chloride, and zinc. The resistance is 1500 ohms. The temperature coefficient is +.000094 instead of the negative coefficient of −.00077 of the Clark cell. After heating to 50° C. the return was nearly but not quite complete. The electromotive force is expressed by the equation $E_t = 1 + .000094\,T$, where the temperature T is reckoned from 15° C.

The only other paper relating to physics read in full at this meeting was that upon "Standards of Length and Mass," by Dr. Mendenhall. This was an announcement of the official acceptance of the international prototypes of the meter and the kilogram recently received by the government. These copies are the result of modern refinement of construction, and are better adapted for use than the earlier standards of length. The result of this action is to make the meter the official standard of the United States, and to lead to the definition of the legal yard in terms of the former.

On the afternoon of April 18 the Academy was received by President Cleveland at the White House. On the following evening it attended a reception at the residence of Alexander Graham Bell. Messrs. Brush, Gould, Langley, Mendenhall, Newcomb, and Remsen were added to the council. No new members were elected at this meeting.

The Annual Meeting of the Institute of Electrical Engineers. — The American Institute of Electrical Engineers held its annual meeting in New York, May 16th and 17th. At the meeting on Tuesday afternoon the election of Professor Houston of Philadelphia as President for the coming year was announced, and various matters of routine business were transacted. In the evening sixty-five members attended a dinner, at which speeches were made by the retiring President, Mr. Sprague, President Houston, Mr. T. D. Lockwood, Mr. T. C. Martin, and others. Wednesday was devoted to the reading of papers.

Following is the programme: —

1. "On the Behavior of Fuse Metals in Direct and Alternate Current Circuits," by Mr. Charles P. Matthews, of Cornell University, Ithaca, N.Y.

2. "A Modified Deprez D'Arsonval Galvanometer," by Lieutenant Charles D. Parkhurst, of Watervliet Arsenal, West Troy, N.Y.

3. "The Variation in Economy of the Steam Engine Due to Variation in Load," by Professor R. C. Carpenter, of Cornell University, Ithaca, N.Y.

4. "An Automatic Printing Speed Counter for Dynamo Shafting," by Professor George S. Moler, of Cornell University, Ithaca, N.Y.

5. "Practical Aspects of Electrical Resonance," by Dr. M. I. Pupin, of Columbia College, New York City.

6. "The World's Electrical Congress of 1893," Discussion of the Provisional Programme prepared by the Institute Committee.

7. "A New Method and Apparatus for Measuring Conductivity," by E. S. Willyoung, of Philadelphia.

8. "The Heating of Armatures," by Messrs. A. H. and C. E. Timmerman, of Cornell University.

The paper which excited the greatest amount of interest was that of Dr. Pupin, of Columbia College. For the purpose of witnessing the demonstrations which accompanied its presentation the Institute adjourned to the Chemical Lecture Room of the School of Mines of Columbia College. Dr. Pupin's experiments upon the tuning of electrical resonators of low period were highly successful, and it was made evident to all who witnessed them that as a means of studying electrical oscillations, the period of which lies within musical limits, the telephone is likely to be of great service. Whereas in the case of the shorter wave-lengths hitherto studied by Hertz and his many followers, the period had to be determined by indirect and at best rather unsatisfactory means, it becomes possible in the case of the range of wave-lengths to which the telephone is sensitive to determine the frequency by ordinary acoustical methods. Those who witnessed Dr. Pupin's demonstrations were struck with the extreme delicacy of the methods which it was the purpose of his paper to exhibit.

It was reported at the close of the meeting that within seven years the membership of the Institute had increased from seventy to nearly seven hundred. The question of extending the usefulness of the Institute by authorizing the local organization of its members in various cities distant from headquarters for purposes of holding meetings was discussed, but no action was taken. Specimens of the new badge adopted by the Institute were exhibited by Mr. Hammer.

NEW BOOKS.

The Theory and Practice of Absolute Measurements in Electricity and Magnetism. Vol. II., Parts I. and II. By ANDREW GRAY, M.A., Professor of Physics in the University College of North Wales. London, Macmillan and Co., 1893. — 8vo. pp. xxiii + 868.

The first volume of this work appeared in 1888. It includes chapters on electrostatic theory, elementary electrodynamics, units and dimensions, general physical measurements, electrometers, the comparison of resistances and of electrostatic capacities, and the measurement of specific inductive capacity. The volume now before us contains chapters on magnetic theory, the determination of H, electric current actions, Lagrange's dynamical method, the electro-magnetic theory of light, the calculation of the constants of coils, galvanometry, the measurement of inductances, units and dimensions, the absolute measurement of resistances, the determination of v, alternating current measurements, the measurement of intense magnetic fields and permeability, and electric oscillations. An appendix giving a brief sketch of spherical harmonic functions is well placed.

The book suffers somewhat from the extensive use of small type, for which, however, the author makes due apology. The printing is good, and the book is provided with an exhaustive table of contents, a good index, and marginal notes, which will add greatly to its usefulness. A greater number of references would be an improvement. In notation, Professor Gray has taken Maxwell's treatise as a basis and has "followed a suggestion of Heaviside in the use of clarendon type instead of German capitals for directed quantities." It will, no doubt, be a matter of regret to many that the condensed notation of the vector analysis has not been made use of in the purely theoretical parts of the subject, for it is almost as much of a task to read carefully a book containing numerous Cartesian formulæ as it is to write it. This is especially true if singularities of notation occur, which, to a certain extent, are inevitable to a work of this kind on account of "the large number of sets of quantities to be symbolized." Professor Gray assumes considerable knowledge on the part of the reader concerning the fundamental phenomena of electricity and magnetism. Still, the absence of terse statements of fundamental phenomena, laws, and conceptions to accompany the formulation of them is to be deplored. No

mathematical treatment can be wholly satisfactory unless so sustained. The cure for a difficulty in mathematical physics cannot be *more mathematics*, except in such developments as partake of the nature of problems as distinct from the development of formal theory. If the difficulties of the student, like those to which Professor Gray refers in the preface, are mathematical difficulties, this work will go a great way towards clearing them up; but if he labor under the more common and serious difficulties which come from weakness in fundamental conceptions, then, to use a term often applied by English critics to German works of this class, Professor Gray's book is surely not sufficiently "metaphysical" to be of much assistance.

It seems that the book could have been made slightly smaller without detracting in the least from its value, by limiting the discussion of such topics as the force action in cavities in magnetized iron to the two important cases; for, in fact, with the exception of just these two cases the whole matter is meaningless. In large cavities the condition of undisturbed magnetization cannot be fulfilled, and in small cavities the conception of uniform magnetization loses its meaning. It is misleading to treat the subject as if our knowledge extended to the inside of a magnet: all magnetic measurements without exception depend upon phenomena which occur outside of iron. Chapter V., on the electromagnetic theory of light, would have been briefer and in some respects more presentable if the symmetrical form of the equations of the electro-magnetic medium due to Heaviside and Hertz had been used. Chapter VI., on the calculation of coil constants, deserves special mention. This is a subject to which Professor Gray has made some valuable contributions. The chapter gives a concise sketch of the subject and gives a number of formulæ which will be of great value to the practical electrician. On the whole, the subject-matter of the book is chosen in accordance with the title; Chapters V. and XIV. (on electric oscillations) being apparently digressions from the plan as announced in the preface to Vol. I. Of no little value and interest are the accounts of experimental investigations, in which the author places "before the reader not merely a skeleton of the method followed and the result arrived at, but such a statement in each case as may serve to show the procedure adopted, the difficulties met with, the mode in which they were overcome, the corrections made, and the reduction of the observations to the final result." Professor Gray has been eminently successful in his "attempt to avoid too sharp a distinction between what is theoretical and what is practical." The main object of his book is the development of absolute measurements, and this object is fully attained.

W. S. FRANKLIN.

Alternating Currents: An Analytical and Graphical Treatment for Students and Engineers. By F. BEDELL, Ph.D., and A. C. CREHORE, Ph.D. New York, The W. J. Johnston Co.— 8vo. pp. 325.

The object of the above work is to give a clear and succinct treatment of the fundamental principles governing the flow of variable or alternating currents; and the authors aim on the one hand to give exact and rigorous demonstrations, and on the other to express the results in such a way as to be perfectly intelligible to those who do not desire to follow the methods of proof, but are only interested in the conclusions reached.

For the task proposed logical order and clear writing were essential; we find them on every page. Thorough knowledge of the subject was essential; the authors are contributors to its original literature. In short, it may be said that the work is what the authors claim, and that they have succeeded in realizing their object in an admirable manner.

If the authors had been writing for professional mathematicians, the natural order would perhaps be to begin with the most general theorem, and proceed from it deductively to the special cases; but as they are writing for students and engineers, they adopt the more suitable method of taking first a very special theorem, then a more general, and last of all attacking the most general theorem considered. Thus they treat first of a simple circuit, afterwards of the complex circuit, and introduce the complexity gradually. In each of these main divisions they first consider a circuit having resistance and self-induction, then a circuit having resistance and capacity, and lastly, the general case of a circuit having all three. As the proofs hinge on the solution of differential equations, this mode of presentation gives the reader an education, probably needed, in the meaning and solution of such equations.

The simple circuit is fully treated by the analytical method, and accurately drawn diagrams are provided to illustrate the results obtained. The complex circuit is not treated analytically, but by means of a graphical method applied to generalize the results for a simple circuit. The work is very free from typographical errors, but the manner of printing the equations leaves something to be desired. On account of the wide spacing many of the equations are too long for one line, and not infrequently the division is made at an awkward place; for factors are disassociated from co-factors, and the function from the variable.

There are a few statements which are not sufficiently exact. Thus the square of a polynomial is said to be "equal to the sum of the squares of each term separately plus twice the product of each term by every other term." Here either the "twice" ought to be deleted, or else "following" added at the end. In the earlier part of the book (p. 41, and p. 80),

Fourier's theorem is not stated with sufficient exactness. Thus at p. 41: "Fourier has shown that any single-valued periodic curve may be built up by combining a number of simple sine-curves. Analytically this means that any single-valued periodic function may be expressed as the sum of a series of sine-terms; thus

$$y = f(x) = A \sin \alpha x + B \sin \beta x + C \sin \gamma x + \cdots \text{etc.},$$

where f is a single-valued function. This is true for any single-valued periodic function, even one represented by an irregular series of straight lines." Now the exact statement of the theorem is: If $f(x)$ is any single-valued periodic function of x of period p, then

$$f(x) = A_0 + A_1 \cos \frac{2\pi}{p} x + A_2 \cos 2 \frac{2\pi}{p} x + \cdots + A_i \cos i \frac{2\pi}{p} x + \cdots$$
$$+ B_1 \sin \frac{2\pi}{p} x + B_2 \sin 2 \frac{2\pi}{p} x + \cdots + B_i \sin i \frac{2\pi}{p} x + \cdots$$

Each A and B coefficient is independent of x, and has one and only one value; and Fourier shows how to find the value. The several arcs are not arbitrary; they are the integer multiples of the first; in other words, the successive periods have the relations

$$1, \quad \tfrac{1}{2}, \quad \tfrac{1}{3}, \quad \tfrac{1}{4}, \quad \tfrac{1}{5}, \text{ etc.}$$

At p. 89 and p. 157 this relation is mentioned, but it is not stated with sufficient prominence.

When a circuit has a resistance R, self-induction L, and capacity C, the discharging current i at any time t is given by the equation

$$i = c_1 \epsilon^{\frac{-RC+\sqrt{R^2C^2-4LC}}{2LC} \cdot t} + c_2 \epsilon^{\frac{-RC-\sqrt{R^2C^2-4LC}}{2LC} t},$$

where c_1 and c_2 are the arbitrary constants of integration. Now if R^2C is greater than L, the expression $\sqrt{R^2C^2 - 4LC}$ is real; but if R^2C is less than L, it is imaginary. Nevertheless there is physical continuity in the change of the character of the current; in the former case the current is non-oscillatory, in the latter it is oscillatory, and in the bounding case of R^2C being equal to L it has an intermediate character. Now this being so, it follows that $\sqrt{-1}\sqrt{4LC - R^2C^2}$ is physically real, being the complement of $\sqrt{R^2C^2 - 4LC}$. The geometrical meaning of this is that the tangent to a circle is the complementary continuation of the ordinate, the latter being $\sqrt{a^2 - x^2}$, the former $\sqrt{-1}\sqrt{x^2 - a^2}$. The authors transform the imaginary expression to a real form in the usual manner — by what

Fleming, in "The Alternate Current Transformer," calls 'a well-known algebraic device' — namely,

$$\sin\theta = \frac{e^{\sqrt{-1}\theta} - e^{-\sqrt{-1}\theta}}{2\sqrt{-1}} \quad \text{and} \quad \cos\theta = \frac{e^{\sqrt{-1}\theta} + e^{-\sqrt{-1}\theta}}{2}.$$

No real student of physics feels particularly satisfied when an algebraic device is presented to him for a reason; he wishes to reason not darkly, but by evident principles. Any one who wishes to see an explanation founded entirely on geometrical principles will find such explanation in a paper which I have recently published on "The Imaginary of Algebra."

At the beginning of "Part II., Graphical Treatment," the authors explain why analytical treatment is not applied to complex circuits. "The analytical solutions derived in Part I. apply merely to a single circuit having resistance, self-induction, and capacity in series. The problems which arise in case there is not simply a single circuit but a complicated network of conductors might be treated analytically, though the process would be exceedingly laborious and the results too cumbersome to handle. Fortunately, however, by making use of the analytical solutions already given in Part I. and extending them by graphical methods, we are enabled to solve problems with compound circuits which offer considerable difficulty to analytical investigation. These graphical methods are most easily and advantageously adapted to problems in which we deal with an harmonic impressed E.M.F."

The principle of the graphical method is to represent a simple harmonic motion by means of the vector to the point describing the auxiliary circle; for a simple harmonic motion is uniform circular motion resolved along a particular diameter. Hence one may reason on the relations of these vectors instead of the algebraic displacements along a diameter. The reason why the ordinary analysis is here found too complex, laborious, and unwieldy, is because it is adapted to quantities which may be represented on a straight line, not to quantities in a plane, still less to quantities in space. But there is a generalized algebra which deals with quantities in a plane, and more generally in space; it is the analytical expression of this same graphical method, is equally direct and clear, but, being a generalization of ordinary analysis, is much more powerful than any graphical method.

ALEXANDER MACFARLANE.

William Gilbert of Colchester, on the Loadstone and Magnetic Bodies, etc. A translation by P. FLEURY MOTTELAY. New York, John Wiley & Sons, 1893. — 8vo, pp. liv + 368.

The need of an English translation of Gilbert's "De Magnete" has long been felt. In this country especially, where copies of the original are

rare and widely scattered, the difficulties that have stood in the way of consulting this most interesting book have been a matter of great regret to physicists and electricians. The appearance, in so attractive a form, of Mr. Mottelay's translation will therefore be hailed with satisfaction by all who are interested in the early history of physical science.

In addition to the text of the London edition of the year 1600, which is translated in full, the volume contains the laudatory address of Edward Wright "to the most learned Mr. William Gilbert, . . . the father of magnetic philosophy," together with a biographical memoir by Mr. Mottelay. The latter is necessarily brief on account of the slight knowledge of the great philosopher that has come down to us.

The annotations throughout the text are numerous, and show the result of Mr. Mottelay's wide acquaintance with early scientific literature. For those who wish to make a critical study of the book they cannot fail to prove of great assistance, while for the general reader they serve in many cases to explain obscure references which would otherwise be unintelligible. It is unfortunate, however, that in a few cases the notes themselves are capable of being misunderstood. For example, on page 117 the reference to Porta is explained in a footnote by a quotation from Porta's "Natural Magick," published in 1658. Since the last authentic edition of "De Magnete" appeared in 1633 the quotation serves rather to confuse the reader than to help him. In this case, as in several others of a similar kind, a few words of explanation would probably make the matter clear.

From an artistic standpoint the book is deserving of high praise. Print, paper, and binding are alike excellent, and far superior to what is ordinarily seen in scientific books. Among the features which add to the attractiveness of the volume may be mentioned the reproduction of the title-page of the edition of 1600, and the use of the Gilbert coat of arms on the cover. At the end of the book the title-pages of the editions of 1628 and 1633, and that of Gilbert's "Philosophia Nova" (1651) are also shown. The numerous illustrations that appear throughout the text are reproduced by photographic processes from the original edition.

The translation appears to be accurate and as nearly literal as is consistent with good English and a correct interpretation of the author's meaning. In a careful comparison of some five or six chapters, taken at random through the book, with the original, I have found but three errors that are worthy of notice. On page 106, line 10, the word "magnetic" should be replaced by "electric." In the first sentence on page 105 I doubt if the full significance of "commouentem" has been brought out in the translation. On page 231 the word "only" should be inserted after "not" in the second line from the bottom, so that the sentence shall read: "In southern latitudes also, and at sea, far away from the

equator and toward the Antarctic, and not *only* in northern latitudes near those magnetic mountains, is variation frequent and great." Although not free from faults, the translation will, I think, prove satisfactory to the great majority of readers. Mr. Mottelay is especially to be congratulated on his success in retaining in his translation some of the quaintness of style which is so attractive in the original and so appropriate to the time at which the book was written. ERNEST MERRITT.

The Voltaic Cell. By PARK BENJAMIN, Ph.D. New York, John Wiley & Sons. — 8vo. pp. iv + 562.

According to the preface the object of this large volume is "to assist the student and investigator . . . by placing before him the record of the principal discoveries and researches relating to chemical generators of electricity." The author further remarks that "the number of sources laid under contribution is large. Chief among them is Dr. D. Tommasi's recent treatise 'Les Piles Électriques,' from which copious extracts have been translated."

Dr. Tommasi's book was published in 1889. It is a large encyclopedic volume of 680 pages, relating to a great variety of combinations which have been proposed as voltaic cells, and containing numerous tables of the numerical results of many investigations relating to electromotive force and the like. The striking resemblance, even at a glance, between Mr. Benjamin's book and that of Tommasi has led me to compare them somewhat at length. The result is to ascertain that of the 535 pages of "The Voltaic Cell," up to the Bibliography, over 300 have been directly translated from Tommasi. These "copious extracts" are taken from ten to fifty pages *en bloc*, with only slight additions interlarded. They include the larger part of Tommasi's tables, which he doubtless collected largely from original sources, and to which Mr. Benjamin has made only a very few additions. So truthful is the translation and so exact the copy that footnotes are frequently included exactly as in the original. The description of cells also very generally follows Tommasi page after page, with an occasional addition drawn from American examples. It is therefore evident that a review of the present volume would be substantially a review of "Les Piles Électriques." According to the literary ethics with which I am familiar, it would have been nearer the facts in the case if Mr. Benjamin had described his book as a translation of Tommasi with additions and an introduction.

The early history of the voltaic cell is well told in the first chapter, and many facts not commonly known are brought out.

The next chapter on definitions is less successful. Volta's invention

is so defined as to comprise a modern secondary battery or accumulator. But it appears somewhat like forcing language to suit one's purpose to speak of "a secondary voltaic cell or voltaic accumulator." This definition scarcely justifies the title of the book, which is made to cover both primary and secondary cells.

Polarization is said to produce "a current in opposite direction to the normal current of the cell." The polarization of an electrolytic cell may produce a current opposite in direction to the one effecting electrolysis, but this is not a correct description of the effect of polarization in a voltaic cell. It is in either case a counter or back electromotive force.

Local action is described as "chemical action occurring within the cell." Of course it is that chemical action which contributes nothing to the energy represented by the current through the cell.

The ohm is incorrectly defined as "the resistance of a column of mercury at 0° C. of 1 sq. mm. section and nearly 105 cm. in length." The length is now well known to be very close to 106.3 cm. So also the electrochemical equivalent of silver is given on page 24 as 1.1340 milligrams per coulomb. It should be 1.118 milligrams per coulomb, or 4.025 grams per hour for a current of one ampère.

The summary of the advantages of sodium bichromate on page 186, taken from my "Primary Batteries," includes one in which I am in error; and this gives me an opportunity to correct it. The sodium salt does not contain a larger proportion of available oxygen than the potassium salt, for the reason that the former has two molecules of water of crystallization.

Among standard cells my own is dismissed with the statement that "the employment of pure chemicals and careful handiwork is claimed to result in a reduction of the temperature coefficient to about .03 of one per cent per degree centigrade." This reduction of the temperature coefficient to .039 per cent is effected in a very different way, and the evidence of it is given numerically in tabular form in my "Primary Batteries," to which Mr. Benjamin makes frequent reference.

The arrangement of materials from Tommasi has been improved by Benjamin, and his bibliography is much superior to that of the French writer. Attention may be called to the fact that the third edition of Hauck's excellent little book on "Die Galvanischen Batterien" appeared in 1890. The 1883 edition catalogued by Benjamin was the second. Also the *Journal de Physique* does not appear among French periodicals bearing on the subject. The whole of Bouty's investigations, quoted from Tommasi, are given in the volume of this journal for 1880.

The book belongs to a class too common in the literature of electricity — books which show that the author is not in touch with the subject by

actual contact with the laboratory or with practical work. Science learned from books and science learned in the laboratory present certain pronounced differences readily apprehended but not easily described.

H. S. CARHART.

Report of Studies of Atmospheric Electricity. By Professor T. C. MENDENHALL. National Academy of Sciences, Vol. V., pp. 113–318.

This monograph, which is a report of attempts made by the United States Signal Service to determine the value of observations upon atmospheric electricity as a means of forecasting the weather, is of great interest to the physicist as well as to the meteorologist. The comprehensive historical sketch of early work upon atmospheric electricity will be welcome to all students in this domain. The discussion of the electrometer, and of the best methods of using that instrument for the measurement of atmospheric potentials, is likewise of great interest.

One cannot but deplore the limitations put upon work of the kind described in this report by the policy of the United States government in such matters. The problem of atmospheric electricity is one of great complexity, and it is one that cannot be solved save by the united action of a large number of observers distributed over wide territory. What precise bearing the results may have upon our knowledge of the weather it is impossible to say, but enough work has been done in Europe and in this country to show that the electrical conditions of the atmosphere play an important part in climatology. It is sad that a government like ours, with unlimited resources at its command, should hesitate to expend the time and money necessary to an exhaustive study of such problems, simply because it cannot be demonstrated in advance that the outcome will be of direct and immediate monetary value.

The instructions under which Professor Mendenhall undertook the direction of this work for the Signal Service were, however, most explicit, and he was confined strictly to the question whether simultaneous observations on atmospheric electricity were of service in forecasting the weather. That no answer to so complex a problem should have been obtained from the comparison of observations extending over two or three years at four or five stations is not surprising. The records which were made and which have been incorporated in this memoir are full of suggestion, and they make it evident that in order to determine ultimately and finally the relation of electrical conditions of the atmosphere to the weather a much greater number of stations should be established, and that the work should be carried on by a great variety of methods. Whether or not such systematic and exhaustive study would result in adding to the certainty of our weather forecasts, the data obtained would be a notable addition to our knowledge

of electricity. The study of electricity for science's sake in the past has found important and far-reaching application. So, doubtless, it would be in this case; but were no results attained, the value of which could be measured in dollars and cents, the United States government, by pursuing work of this description, which, as has already been pointed out, lies beyond the reach of any individual laboratory or institution, would be performing one of those functions which certainly in the future we must look to government to perform. It would thus enable a people to reach knowledge unattainable by individual and unorganized effort. It is to be deplored that these observations were abandoned after so short a trial; and it is to be hoped that one of the results of the divorce of the Signal Service and the Weather Bureau will be to hasten the day when subjects like this may be attacked in a manner in some sense adequate to their solution.

E. L. NICHOLS.

Handbuch der Physik. Herausgegeben von DR. A. WINKELMANN. Breslau, 1893 (Edward Trewendt). — Band 3, I., pp. 546.

Winkelmann's Handbook is one of the two great general treatises on physics now in process of production. Being the work of more than a dozen colaborers, it lacks that unity and conciseness of treatment which characterizes Violle's great treatise; but it exhibits in a striking manner the indefatigable industry characteristic of German men of science. It is especially rich in the two features which render such a work of value to the practical physicist; viz. unusual fulness in the matter of references to original authorities, and a wealth of tabulated material. Theoretical development, on the one hand, and elementary description and explanation, on the other, have wisely been given a secondary place, and the work is chiefly to be regarded as a compendious summary of well-established results.

Volume III., the first part of which has just appeared, is principally from the hands of Dr. Graetz of Munich, and of Professor Auerbach of Vienna, with chapters on the electric conductivity of gases by Professor Staenger of Dresden, and upon thermo-electricty by Professor Braun.

Graetz, after giving a brief development of the potential theory, treats of electro-statics at considerable length, devoting special attention to the properties of the dielectric. The data gathered under this head from a variety of sources will be found to be of great value to workers in the new field of electro-optics.

Current electricity, beginning with contact phenomena, and including a brief treatise upon voltaic cells and methods of measuring current and electromotive force, is treated by Professor Auerbach.

The subject of electrical conductivity has been divided into three parts: the resistance of metallic bodies, by Graetz; of electrolytes, solid and liquid, by the same author; and the conductivity of gases, by Dr. Staenger. Electrolytic conductivity is entirely separated from electrolysis proper, to which an important chapter is given in another portion of the volume.

The third volume of this work is to be devoted entirely to electricity, and Part I., which is now under consideration, forms in itself a book of more than five hundred large pages.

Considering the increasing difficulty of following the results of investigation in the original sources, the appearance of such a compilation as Winkelmann's Handbook will be welcome to physicists, if for no other reason, because it will greatly facilitate acquaintance with the contents of the numerous papers whence the mass of material contained within its covers has been derived, and through the accompanying bibliographical work, with the memoirs themselves.

In a work of this character, especially when it is the joint product of several writers, one does not look for uniformity of treatment. In the present treatise much that has gone to make up the bulk of previous text-books is passed over briefly or is altogether omitted. It is in topics heretofore very inadequately treated that Winkelmann's work is especially strong.

E. L. NICHOLS.

Some Recent Text-Books in Physics.[1] — The first book in the list given below is intended as a text-book for classes in high schools, or schools of a similar grade, where the mathematical preparation of the student is of the most elementary character. For this reason the subject has been presented in such form that only the simplest ideas of geometry and algebra are required, while the only mention of a trigonometric function occurs in the discussion of the tangent galvanometer. The difficulties that are met with in preparing a satisfactory text-book under these restrictions are obvious. In some respects it would be easier to write an elaborate treatise. The authors have been unusually successful, however, in avoiding the faults which are too often met with in books of this class. The statements and explanations are, in

[1] *The Elements of Physics.* By HENRY S. CARHART and HORATIO N. CHUTE. Boston, Allyn and Bacon, 1892. — 8vo. pp. 382.

Discussion of the Precision of Measurements. By SILAS W. HOLMAN. New York, John Wiley & Sons, 1892. — 8vo. pp. 176.

A Student's Manual of a Laboratory Course in Physical Measurements. By WALLACE CLEMENT SABINE. Boston, Ginn & Co., 1893. — 8vo. pp. 120.

A Manual of Physics. By WILLIAM PEDDIE. New York, G. P. Putnam's Sons, 1892. — 8vo. pp. 501.

general, brief but clear. In no case has accuracy been sacrificed for apparent simplicity. Although not free from faults, the book cannot fail to be of assistance in raising the standard of elementary teaching.

Of the different branches of physics that are discussed, the treatment will strike the reader as least satisfactory in the case of electricity and magnetism, although it is difficult to tell just where the fault lies. It is to be remembered, however, that the subject of electricity is probably the most difficult in physics to present in an elementary manner. The omission of all mention of double refraction and polarization in the section on light seems unfortunate. It is doubtless intentional, and probably results from the experience of the authors in the difficulty of teaching these subjects to an elementary class. I cannot help thinking, however, that a few well-written pages devoted to the simpler phenomena of polarization would add to the value of the book.

The numerous illustrative experiments that are described throughout the book will doubtless prove of considerable value to teachers. These experiments are, in most cases, so simple that they could be performed in the class-room, or by the student alone. The illustration of physical laws by reference to the familiar facts of everyday life is also deserving of mention, the sections on heat and sound being in this respect especially satisfactory. There are several cases, however, in the section on mechanics where the explanations and illustrations seem inadequate. The subject of the composition of velocities, for example, might be more clearly presented. I doubt also if the discussion of fluid pressure on pages 90–91 will make the matter entirely clear to a beginner. The most striking omission, however, is in the case of the third law of motion, which is barely stated, and left with practically no explanation. Such an omission is especially to be regretted in a book of this kind, which is unfortunately too often placed in the hands of a teacher whose training is not sufficient to enable him to supply its deficiencies.

Professor Holman's "Discussion of the Precision of Measurements" is intended for the use of students in the laboratory, and presents in a clear and concrete form the usual methods of treating the accidental errors of observation. The book will be found to differ widely, however, from the ordinary text-book of least squares. The results of the theory of probabilities are stated without proof, and serve as a basis for the discussion which follows. It appears to me doubtful whether the omission of proof in the more fundamental cases is advisable. References should at least be given to books where the mathematical treatment can be found. But the discussion of the principles involved is so excellent that criticism on this point is in large part disarmed.

The scope of the book will perhaps be best indicated by the titles of

its four principal divisions: I. Direct Measurements; II. Indirect Measurements; III. Best Magnitude of Components; IV. Solutions of Illustrative Examples. In the first section will be found a general discussion of the constant and variable errors of observation, illustrated by carefully chosen examples. The use of the "average deviation" (the average value of the deviation from the mean, neglecting signs) instead of the "probable error" will meet with the approval of all who have observed the difficulty that is experienced by students in obtaining a proper understanding of the latter term. The second division is devoted to the consideration of cases where the final result is some function, more or less complex, of the quantities directly observed. This chapter would be improved if the treatment were made more brief. The detailed discussion of the special forms that may be assumed by the function which expresses the result seems entirely out of place. In fact the twelve pages that are devoted to this purpose rather detract from the usefulness of the book by giving an appearance of complexity to a subject which is really simple. The examples at the end of the chapter would gain in clearness if they were explained by a direct use of the principles of the method. The reference to special formulæ, which requires the student to retain in his memory an arbitrary system of notation, does not appear to me to be justified in the present case.

The fourth section, which is devoted to the solution of illustrative examples, is in some respects the most valuable in the book. Of the seven problems that are discussed, the following may be mentioned as of especial interest: the calibration of a voltmeter; the measurement of power by the cradle dynamometer; and the discussion of the sources of error in the use of a tangent galvanometer. Apart from their illustrative value the examples discussed in this section will prove useful for reference in the laboratory.

It may not be out of place to call attention to the misprint on page 132, where we find the statement that "1 lb. = 13825 g ergs." So much misunderstanding already exists in regard to the two systems of units that it is unfortunate that any avoidable cause of confusion should be added.

The "Manual of a Laboratory Course in Physical Measurements," by Mr. Sabine, represents a grade of work that is intermediate in character between the course which would be expected in a preparatory school and that which is usually required in colleges. The elementary character of the book is doubtless explained by the fact that the course, as given by Mr. Sabine, is intended to replace the experimental lectures which in most colleges are made to precede work in the laboratory. Although the desirability of this arrangement of work appears to me very questionable, it would be out of place to discuss the subject here. For the purposes in view the book is an excellent one.

It is the intention of the author that the experiments should be performed in the order in which they occur in the book, the class being broken into sections if necessary, and working under the direction of an instructor. To render this plan practicable without an excessive outlay for equipment, the experiments have been so planned as to require only the simplest apparatus. The reader cannot fail to be impressed with Mr. Sabine's success in this part of his task. A danger that arises, however, from the use of very simple apparatus lies in the fact that the student is apt to look upon his experiments as merely illustrative. Careless habits of observing result, and although at the end of his course the student may have obtained a fair understanding of physical phenomena, he has failed entirely to acquire a grasp of the quantitative methods of the laboratory. It is in the lack of sufficient precautions against this tendency that I consider the book chiefly in fault. The general directions in the introduction are excellent, while the explanations of the principles involved in the various experiments are unusually clear. But in the discussion of the sources of error, and of the methods to be used in determining the corresponding corrections, the book is uniformly unsatisfactory. The value of the course would be greatly increased if the treatment were made more rigorous, even if this should require a considerable reduction in the number of experiments.

Although the book is in general free from errors, two instances may be mentioned where the statements are not above criticism. On page 45 the author states that "theoretically mercury increases in volume as much when the temperature is changed from 0° to 50° as when it is changed from 50° to 100°." The word "theoretically" seems to be inserted in order to indicate that the statement is not strictly true. Although the use of the term in this sense is not uncommon, it is certainly not to be commended. The use of the gram as the unit of force on page 23 is unfortunate in view of the statement made in the introduction that the C. G. S. system would be adhered to. This error will probably correct itself, however, since the dyne is defined and correctly used on the following page.

The "Manual of Physics" by Dr. Peddie can scarcely be called a text-book in the sense in which the term is ordinarily used. As stated in the preface, the book is intended to give a review of the whole subject of physics for the use of students in the universities. For this purpose it has many advantages. The treatment throughout the book is extremely suggestive, while the amount of valuable matter that is contained in its 500 octavo pages can scarcely be realized until the book is read. The evil effects of crowding are noticeable, however, throughout. There is an indescribable something that suggests an approaching examination, and which cannot fail to give to the reader a feeling of hurry and unrest.

The mathematical treatment, especially in the early part of the book, presents some novel features. The discussion of the theory of contours in Chapter III., and the use of the simpler methods of vector analysis in Chapter V., will meet with approval, as indicating the advances that are being made in the application of graphical and vector methods to physics. In both cases, however, the discussion will prove satisfactory only to those who have studied these subjects elsewhere. For a student who is approaching the matter for the first time the treatment is quite inadequate. This remark will apply with equal force to most of the mathematical work in the early part of the book. Chapter IV., which is devoted to the explanation of the elementary processes of differentiation and integration, seems out of place in a manual of physics, and is insufficient to prepare the student for the applications of the calculus which follow. The elementary explanation of the use of Cartesian co-ordinates in Chapter V. seems also unnecessary.

In what might be called the philosophy of physics the book is strong. The discussion of the uses of theories and hypotheses in physical science is good, although the author weakens his position by the bold statements that "as knowledge advances the truth of some theories will be proven," and that "an elementary molecule or atom has absolutely unalterable properties." The parallel discussion of the undulatory and corpuscular theories in the section on light is instructive, and affords an opportunity for illustrating the methods of discriminating between theories which at the outset appear equally plausible. On the whole the book seems to be well suited for the purpose set forth in the preface. I doubt, however, if it will prove of value as a text-book.

ERNEST MERRITT.